Ionospheric Research

NASA Grant No. NsG-134-61

Scientific Report

on

"System Optimization for a Large Information Collection System"

by

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March 10, 1967

Scientific Report No. 292

Ionosphere Research Laboratory

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ABSTRACT

This work does an investigation of large data collection systems and in particular the processes which result in lost information. A comparison is made of the sensitivity of different system configurations to information loss by overloading the receiving or serving mechanisms with random signal arrivals, and errors of symbol interpretation due to propagation through a noisy and fading channel. The channel is assumed to have additive, white Gaussian noise and a Rician fading structure.

The types of systems operation examined include sequential interrogation of the data sources, and random arrivals with or without signal separation capabilities. The servicing mechanism is examined for information lost considering such factors as arrival rate, number of receivers, independent or dependent operation, and the number of redundant data periods per transmission for each source.

I. INTRODUCTION

With the significant increase in modern computational facilities, data collection systems on a very large scale are becoming increasingly common. Large computational, storage, compilation, retrieval, and analysis capabilities permits the handling of data from collection systems having a large number of sources. The mobility of a satellite allows a collection device which can cover large geographical areas. These systems can be used to provide a quick reaction response to oceanographic, seismic, or atmospheric disturbances, or they may be used for gathering background data from which models of physical phenomena can be made and analyzed for long term prediction and understanding.

There appears to be a need for work relating the various relevant factors affecting the system performance of large scale information collection systems. Several such systems are in the planning stage and it is possible that expansion of existing systems might be considered in other cases.

A great deal of the work presently being done is related to power consumption, spectrum usage, receiver sensitivity, and other practical considerations. These, of course, are most important because the feasibility of the system must precede evaluation of system performance. However, granted feasibility, attention then turns to optimization considerations of systems design or systems philosophy. Many questions arise concerning the optimum number of sources or receivers, the maximum allowable bandwidth and readout time, and the minimum acceptable rate of information loss. These are the types of problems to which this study has been directed.

While these collection systems may differ in nature, there are several similar problems and characteristics. Some environmental parameters are monitored by a group of sensors, and the state of the environment in that locality, upon conversion to a recognizable language, becomes the raw data of the collection system. The data are then converted to a form suitable for transfer through a communication channel. Arrival of the data from each of the sources at the processing or reception mechanism completes the transfer phase of the data, and initiates the processing or servicing phase of the collection system.

The servicing function can be accomplished in several ways depending upon the nature of the individual arrivals. First, the arrivals could be time separable by having the sources respond sequentially to an interrogation signal. Secondly, the number of arrivals in an interval could be subject to a random distribution with or without the additional capability of signal separation by frequency, location, or signal orthogonality. In the first, information is not lost by the servicing operation. However, in the second, because of the random nature of the arrivals, overloading within a particular interval can occur and this can directly cause an arrival to be lost or incompletely processed, and hence, information is lost.

It is, then, the intention of this study to examine the two stochastic processes that are associated with lost information. The one relating to reception system overloading and the other to symbol errors due to data transfer in a noisy, fading communication channel.

II. HISTORICAL BACKGROUND

The problem studied was suggested to the author by Dr. John S. Nisbet of The Pennsylvania State University. It arises from oceanographic and meteorological reporting systems presently under consideration such as those described by O'Rourke (1965) and The National Academy of Sciences (1966). Most of the previous work relates to system feasibility. Once feasibility has been demonstrated, the next logical step is to consider system design criteria and performance optimization. This study anticipates the need for research in that direction.

The research reported here does not use methods that can be easily classified under one descriptive branch of electrical engineering. However, the approach and goals may be associated with what has been generally understood by the terms systems analysis or systems engineering. Included in this treatment are elements of statistical communications theory, stochastic processes, and queueing theory.

Considerable attention has been given to optimal design of communications systems and systems concepts since World War II, for example, see Kalaba and Juncosa (1956) and University of Chicago (1957). By 1945, it was generally accepted that studies of communications systems could no longer ignore noise effects, and most studies since then have treated both the pure signal and the effects of the noise. A paper by S. O. Rice appeared in 1945 and then became a foundation and stimulus for later investigations. This paper mathematically treated the effects of noise in physical systems.

For telemetry systems using pulsed signals, Van Vleck and Middleton (1946) have discussed reception with only a limited knowledge of the received signal characteristics. The decisions made by the receiver system logic were accomplished by statistical inference.

Other investigators have made fundamental contributions to the general theory, including stochastic processes, Weiner (1949), optimum filters, Zadeh and Ragazzini (1952), and matched filters (filters whose frequency response matches the frequency spectrum of the signal), Van Vleck and Middleton (1946), and Turin (1960). Several comprehensive textbooks have appeared on the subject of statistical problems of communication. Perhaps among the most notable of these are Middleton (1960), Lee (1960), Davenport and Root (1958), and Wainstein and Zubakov (1962).

In 1955, Helstrom (1955) considered the resolution of two signals in white Gaussian noise, and later Turin (1956) used these results to find the binary error probability of noisy multiple channels with Rician fading. Lindsay (1964) considered multiple channel fading problems with an N-ary communications code.

The purpose of that part of the study related to systems error probability is to examine the system performance under the effects of signal to noise ratio, error probability, the symbol interval, and the parameters of fading. In this study, the system symbol recognition error as a function of signal to noise ratio, symbol time, and other factors, is assumed independent of the over-all criteria and is utilized separately.

The first theoretical research into the properties of queues began with problems of telephone operation, and the most notable of these is the work of Erlang (1918). A queue is defined as a waiting line similar to the line formed by patrons at a ticket counter. Erlang's work stimulated other investigators such as Fry (1928), whose book deals with a wider class of queueing problems. Other pioneering works in the theory of queues are those of Pollaczek (1930) and Khintchine (1932). More recently, research by Kendall (1954) formed the basis for most analysis techniques which utilize the inherent Markov properties found in these processes. This is known as the imbedded Markov chain. Lindley (1952) provided an integral equation approach to queues with only a single server. Notable books treating the general theory of queues have been appearing regularly. Some of these are Morse (1958), Saaty (1961), and Takacs (1962).

In 1932, Crommelin (1932) derived waiting time equations for a queue with fixed service times if there was no limit to how long a call could wait in line. Then Everett (1953) found the probabilities of being in each state for waiting lines with a fixed service time. More recently, Burnett, Bogar, and Konhauser (1959) considered both multiple and single server queueing problems for various service times. They found expressions for the waiting time and mean waiting time. Later Daley (1964) obtained a general solution to the single server queue with a fixed time allowed for waiting.

In the study reported in this thesis, we will be concerned with a finite waiting limit and a constant service time, or with no waiting and multiple servers.

Much of the work done previously assumes that the service times follow a negative exponential probability distribution. While this is useful in telephone applications, we are concerned here with telemetry type arrivals or calls where the service interval is normally a controlled fixed length. Also a large measure of previous work is concerned with expected waiting times. We are interested in the probability of either no service or an incomplete servicing, both implying lost information.

The information lost by the servicing of arrivals, in conjunction with the lost information from misinterpreting the received symbol, forms a part of the operational characteristics to which system optimization techniques can be applied.

III. STATEMENT OF THE PROBLEM

3.1. General Statement

The basic problem area studied is related to the optimal design of information collection systems where there are a large number of data sources. The study attempts to develop the necessary relationships required for making systems design and philosophy decisions. The systems considered will generally have noise and fading problems and separation of the sources by time, frequency, space, or other means may not always be possible.

3.2. Specific Statement

This study is confined to the context of a large scale collection system without being specifically bound to a particular model or application. The two stochastic processes which will affect the rate of information loss of the system will be examined. The two loss processes are the information loss resulting from misinterpreting the transmitted symbol and the loss resulting from overloading the servicing mechanism.

The study attempts to examine the system performance associated with each of the following considerations:

1) the symbol interpretation error associated with transmission in a noisy, fading channel;
2) the probability of information being lost or incompletely processed due to overloading the receiving system;

2a) the effect of data redundancy on information loss;

2b) the effect of additional independent receivers on the information loss;

2c) the effect of additional dependent receivers on the information loss;

3) the total information loss from both symbol errors and servicing;

4) the time-bandwidth problem associated with different types of system operation;

4a) the sources are sequentially interrogated without time coincidence;

4b) random operation of the sources with no separability capability;

4c) random operation of the sources with separability possible.

The intention of this study was not to become involved with a specific design of all or any part of the collection system, but to assume general feasibility and then provide the data for making decisions pertaining to systems philosophy. Nor was it the intent of this study to be confined to any particular model or application.

IV. TYPES OF SYSTEMS AND SYSTEMS OPERATIONS CONSIDERED

In trying to obtain a satisfactory systems concept for a large scale data collection system, one of the first problems encountered is the problem of how to handle the large flow of data from the sources to the servicing mechanism.

4.1. Sequential Interrogation

One of the first methods that is usually considered is to sequentially interrogate each source. The interrogation device operates in conjunction with the processing device so that unique identification is obtained. The interrogation signal must carry a coded signal uniquely recognizable by only one data source. After recognizing the code, the source then transmits its data relating to the present or past stored state of the sampled environment. The receiver, upon receiving the transmission without interference from other sources, can identify the source uniquely because of the coded interrogation. This type of operation has one great advantage. There is no interference from other sources during the transmission, and, hence, there is no lost information from overloading the server. There are three disadvantages of such an operation. First, each source must have a capability to receive, decode, and identify a signal sent by the interrogator. Such a capability may prove too costly if the number of sources is large. Secondly, since the readout of each source is time sequential, the total time to read out the whole system may be excessive and could exceed the limits for a quick reaction capability. The last disadvantage is the necessity for providing the interrogation capability and a communication link with the servicing receivers for identification purposes. If the interrogation device is a satellite, the additional requirements of antennas, transmitter, directional stability, and power requirements might be undesirable.

4.2. Random Arrivals without Separability

Another type of operation that has some utility is to cause the sources to transmit randomly in time for a complete data interval duration corresponding to a time multiplex transmission for every environmental parameter monitored. The data interval is thus fixed for all sources. Since there is no separability of signals, a unique identification code for each source must also be transmitted along with the data. The sources might start transmission randomly corresponding to a mutual probability distribution, or they might be associated with a fixed incremental change of each or some specific environmental parameter monitored. This type of operation has the advantage of requiring no interrogator and no communication link between servicing and interrogation, and the individual data sources do not require a reception capability. The disadvantage of such a system is that without the separability capability of the signal, the arrival rate of the signals at the processing receiver must be sufficiently low to prevent time overlapping of data signals from different sources. Since this cannot be assured with certainty, it must be accomplished only for an acceptable rate. This means, that by reducing the signal arrival rate sufficiently, time overlapping which causes lost information can be reduced to a satisfactory level. Unfortunately, the reduction in arrival rate usually leads to an excessive readout time for the whole system. The problem then reduces to a balancing of arrival rate, probability of lost information, and readout time. If upper limits are placed on two of these, it may not be possible to adjust the third to satisfy the restrictions.

4.3. Random Arrivals with Separability

This type of operation differs from the former in that we assume that separation of the signals can be accomplished by other means than transmission time such as by the use of frequency, space, orthogonal signals, etc. Therefore, information is not necessarily lost if time overlapping occurs as in previous operations.

1. Multiple server operation

This type of operation allows the potential utilization of additional servicing receivers by the use of a scanning master receiver and slave receivers capable of being assigned to service a specific signal. Naturally, since time overlapping is possible, the probability of lost information due to an arrival encountering an already busy server can be reduced by increasing the number of servicing receivers.

2. Redundant data signals

An alternate type of operation that is quite interesting is the use of data signal redundancy by causing the complete data interval to be repeated several times in one transmission. This, in effect, forms a uniformly finite duration queue with reneging (leaving a queue before service is completed). If the signal has not started service before the last repeated data interval, the signal is lost or incompletely processed. The use of repeated data intervals, however, allows the signal to "wait" for a server to become free, and the signal can be processed on a subsequent data interval instead of being lost immediately if the server is occupied upon arrival.

The advantage of multiple receiver or data redundancy operation is, of course, that the arrival rate can be made larger by the addition of receivers or data intervals. This implies that the readout time and lost information rate can be reduced correspondingly. The disadvantage of such operation is the cost of additional receivers or data intervals and the cost of the separability capability. If frequency separability is used, the system bandwidth may be excessive. If space separability is used, the associated system problems which result from the need for high resolution antennas may prove to be insurmountable or cost too much to achieve. Signal orthogonality would require more sophisticated correlation reception.

Of course more sophisticated operations and combinations of the different techniques mentioned are possible. Those considered form a basis for a study relating to first principles of operation, and further examples can be treated by techniques similar to those developed in this study.

V. APPLICATIONS

Several examples of practical applications consistent with this study are the following:

1. A traffic density and classification monitoring system for a large city freeway, bypass, or turnpike system. A large number of traffic classification and density monitors could be located at strategic points. The information as to vehicular type, rates, and volume could be transmitted to the collection device by telephone wires or by radio signals. The data could be analyzed to control or synchronize traffic signals or to specify alternate routings for cars, trucks, buses, emergency vehicles, etc.

2. A weather reporting system capable of making measurements of temperature, pressure, wind velocity and direction, humidity, etc. The sensor-transmitter stations might be small unattended units scattered over uninhabited regions of the earth, fixed or floating at sea, or attached to tethered or free floating balloons. The data could be gathered by a central processing center such as a satellite or ground station. The data could be analyzed for quick reaction warning of potentially dangerous weather situations or it could be used in making meteorological models for understanding and study and long term prediction.

3. A reconnaissance system whose purpose it is to monitor radar, radio, or telemetry signals associated with scientific, economic,

political, or military activities of some otherwise inaccessible targets. In such an application, control of the sources by the collection device is generally not possible.

VI. PROBABILITY OF LOST INFORMATION, I

Overloading the Collection System

The first problem we shall treat will be the probability of information loss due to receiver overloading. In the case of sequential interrogation, there is no receiver overloading, and, hence, no loss of information by this mechanism.

6.1. Random Arrivals without Separability

In the case of random arrivals without separability, loss can occur when two or more sources are transmitting simultaneously.

First, we shall make the assumption that the number of arrivals in any time interval is independent of any other non-overlapping time interval and depends only on the length of the interval. Let λ be the mean number of arrivals per unit time which is assumed to remain constant. This leads to the Poisson Distribution and following Saaty (1959) the probability of exactly n arrivals during x time units is

$$P_n(x) = \frac{(\lambda x)^n e^{-\lambda x}}{n!} \quad . \tag{6.1.1}$$

The probability density $\tau_n(x)$ for exactly n arrivals in exactly x time units is

$$\tau_n(x) = \lim_{\Delta x \to 0} \frac{(\text{probability of } n - 1 \text{ arrivals in } x)(\text{probability of one arrival in } \Delta x)}{\Delta x} ,$$

or

$$\tau_n(x) = \lim_{\Delta x \to o} \frac{\dfrac{(\lambda x)^{n-1}\, e^{-\lambda x}}{n-1!} \quad \dfrac{\lambda \Delta x\ e^{-\lambda \Delta x}}{1!}}{\Delta x} \,. \tag{6.1.2}$$

This reduces to

$$\tau_n(x) = \frac{\lambda(\lambda x)^{n-1}\, e^{-\lambda x}}{n-1!} \,. \tag{6.1.3}$$

In the non-separable random arrival situation, information can be lost if upon the arrival of each signal another signal is being serviced, or if during service another arrival occurs. The probability of information loss for this case is P_1 and may be described by,

> P_1 = the joint probability of an idle server upon arrival and at least one other arrival occurs before service is completed plus the probability of a busy server upon arrival of the signal.

This becomes

$$P_1 = \left[1 - P_o(T_s)\right] \int_{T_s}^{\infty} \tau_1(x)dx + \int_{o}^{T_s} \tau_1(x)dx \tag{6.1.4}$$

or

$$P_1 = \left[1 - e^{-\lambda T_s}\right] \int_{T_s}^{\infty} \lambda e^{-\lambda x} dx + \int_{o}^{T_s} \lambda e^{-\lambda x} dx. \qquad (6.1.5)$$

After integration P_1 becomes

$$P_1 = 1 - e^{-2\lambda T_s}. \qquad (6.1.6)$$

From equation 6.1.6 it is apparent that P_1 is just the probability that there is at least one arrival either within the interval T_s before or within the interval T_s after the arrival considered, since $P_o(2T_s) = e^{-2\lambda\ T_s}$ and $1 - P_o(2T_s) = P_1$. Equation 6.1.3 is shown graphically in Figure 1. It can be seen in Figure 1 that a low probability of loss (below .1) may be obtained only for a low value of λT_s (.05 or less). In this region, P_1 may be approximated by

$$P_1 \doteq 2\lambda T_s \qquad \text{for} \qquad \lambda T_s \leq .05. \qquad (6.1.7)$$

Figure 1 indicates that this method of service will not be very useful if the readout time, which depends on λ, and the probability of loss are desired to be as small as possible. If one is restricted to a specific value, it is possible that the other can not be adjusted to obtain satisfactory results. It should be noted that this type of servicing results in lost information from both the signal being

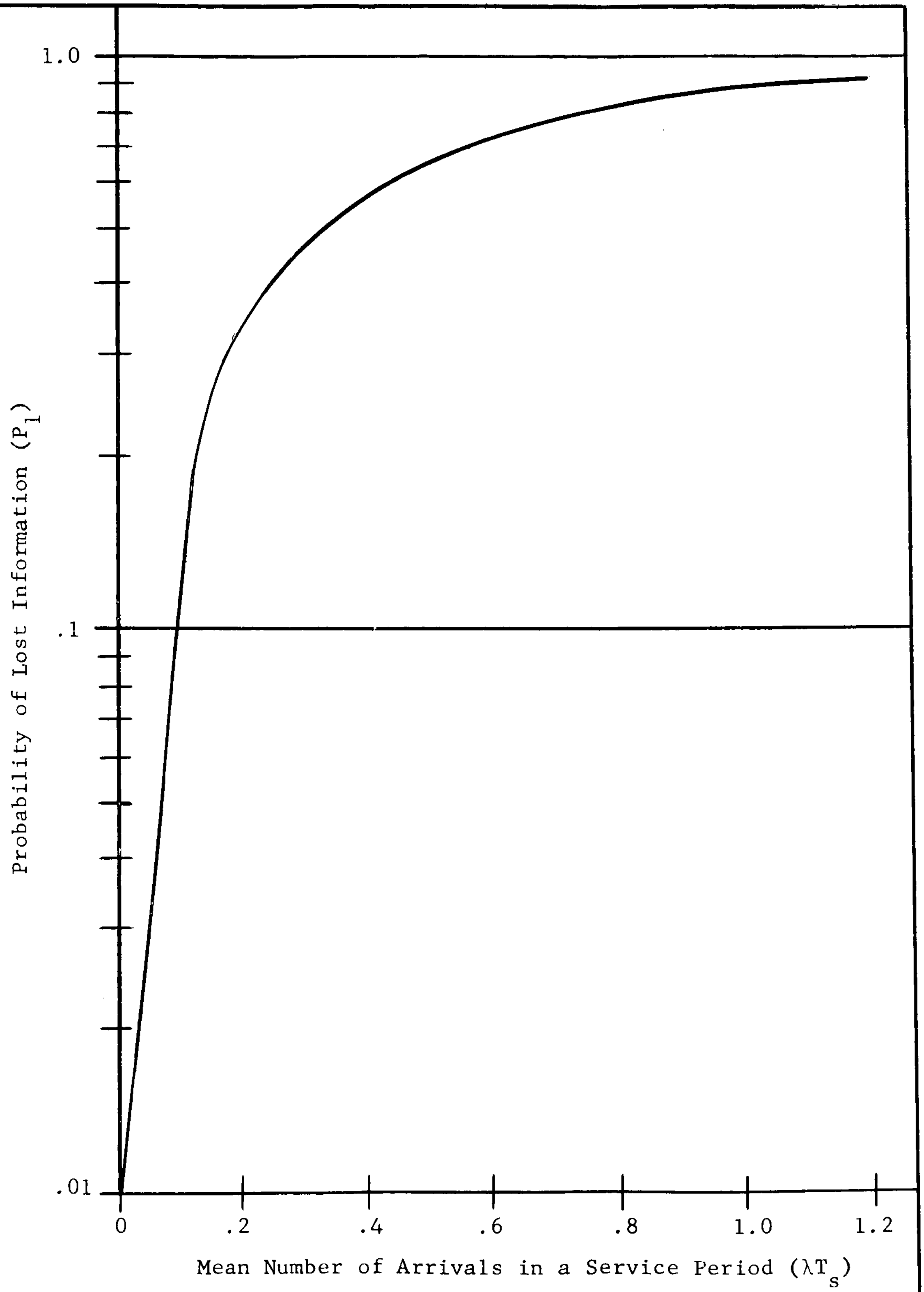

Figure 1. Probability of Lost Information for Each Arrival with Non-Separable Sources.

serviced and the signal which arrives during that service. In later servicing models, only the signal which arrives during the service period will contribute to the total loss.

6.2. Random Arrivals with Separability

Consider two methods of decreasing the probability of information loss. In the first, the servicing function is performed by multiple receivers operating in such a way that each receiver may service a different arrival. In the second, data redundancy is obtained by repeating the data several times during each transmission period. Once the data has been serviced, the receiver is free to be reassigned. The second method is discussed later.

1. Multiple receivers

Three cases of multiple receiver operation will be considered in order of increasing complexity. In the first, signals are assigned successively to receivers as they become free. In this case, every arrival will start service sometime within its duration. However, that service may be incomplete if the service did not start immediately upon the arrival of the signal. This type of servicing permits a signal to be incompletely processed, but does not effect the signal being serviced if another arrival occurs before that service is ended, either by completion of service or by termination of the signal.

In the second system, the signals are assigned with respect to the order of arrival in rotation to the several receivers. The new signal is lost or neglected if the server to which it is assigned is busy upon its arrival. This has the advantage over the previous case, in that the server is not occupied with signals it can not service completely.

Case 1. All calls start service—In this system all calls are assigned in order of arrival. It makes no difference in this case if they are assigned in rotation to receivers or to the receiver which is longest in service if none are free. If some are free, it may be assigned to any free one. All calls will be serviced for either a portion of a service interval or completely. None are lost or ignored. However, when the server becomes free, it then begins service on the new arrival whether or not a complete service can be made before the signal terminates. In any event, if a server is occupied with a signal when a new arrival appears, the signal in service is not affected. This was not true for the non-separable case.

For all calls starting service, the probability that an arrival will be lost is the probability that all receivers are busy when the arrival occurs. This may be recognized if we note that as the remaining portions of each signal are processed, the first to become free is the one that was first assigned.

Let c be the number of receivers or servers. The probability that all c receivers are busy upon the occurrence of a new arrival is given by

$$P_2 = \int_0^{T_s} \tau_c (x)dx. \qquad (6.2.1)$$

Using 6.1.3 for c arrivals we have

$$P_2 = \int_{o}^{T_s} \frac{\lambda\ (\lambda x)^{c-1}\ e^{-\lambda x}}{c-1!}\ dx. \qquad (6.2.2)$$

This may be evaluated as

$$P_2 = 1 - e^{-\lambda T_s} \sum_{i=o}^{c-1} \frac{(\lambda T_s)^i}{i!} . \qquad (6.2.3)$$

The effect of the number of receivers and the mean number of arrivals in one service period (λT_s) on the probability of an incomplete service is shown in Figure 2. The continuous curves were calculated from 6.2.3 and the indicated data points are the results of a computer simulation of the all calls starting service model. The agreement between analytical and simulated results is shown to be quite close. In the lower values of probability, the number of lost arrivals for the simulation was small and larger variances resulted.

Case 2. Independent receivers with delayed arrivals lost—In the second case, the new arrivals are assigned in rotation to the receivers on the basis of order of arrival. If the receiver to which it is assigned is occupied upon its arrival, the arrival is lost or ignored by the servicing system. The next signal is assigned to the following

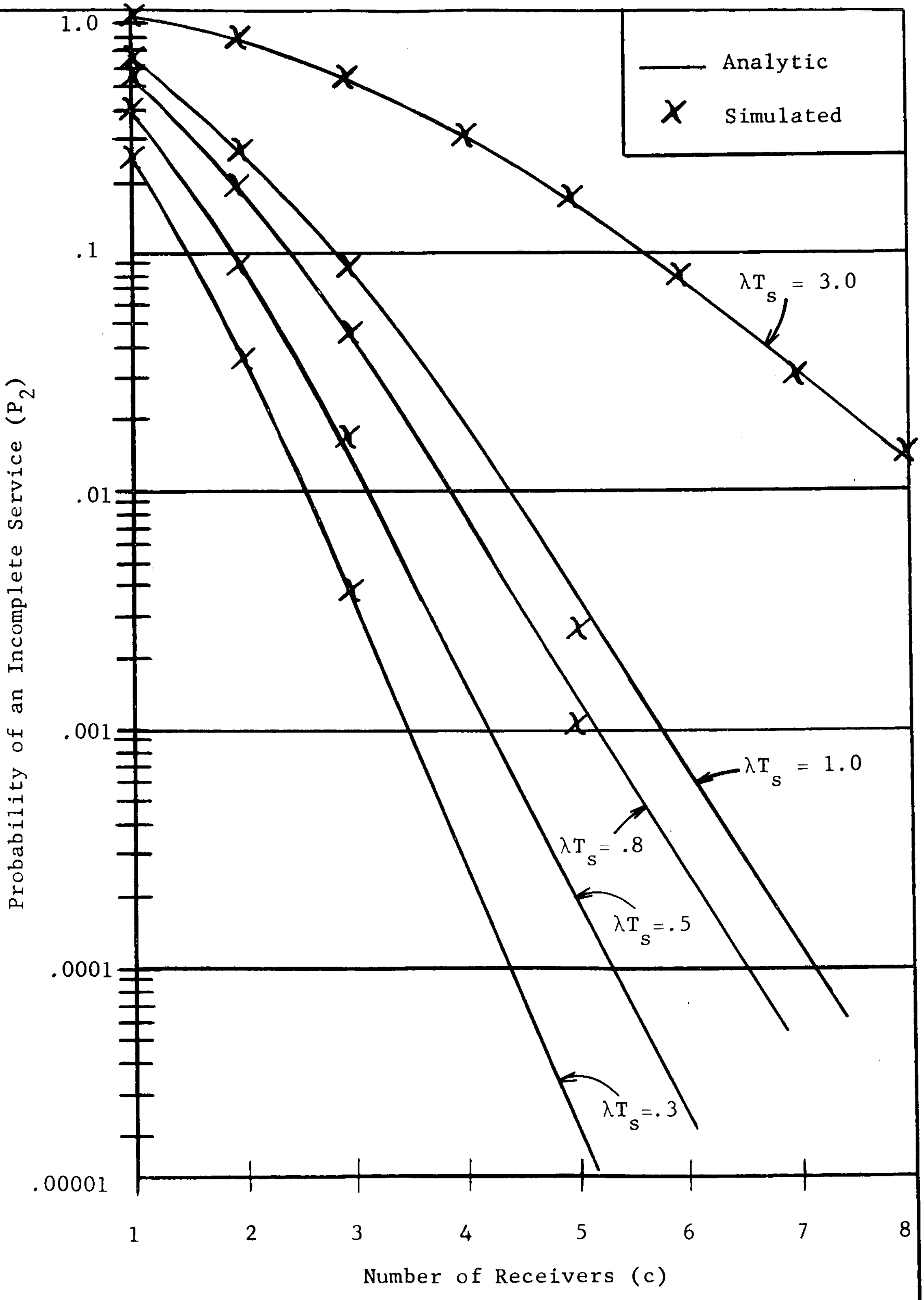

Figure 2. Probability of an Incomplete Service for Multiple Receivers when All Arrivals Start Service.

receiver if it is free and similarly rejected if it is occupied. This system has the advantage over the previous system considered in that incompletely serviced calls will not occur and hence can not block a subsequent arrival.

The probability that a call is not processed is in this case more complex because it depends on the probability that previous calls assigned to that server have not been processed. Let the probability that a call is not processed be P_3. Then we shall first require the probability $p(x)dx$ that the last arrival occurred at $x + dx$ if service started at $x = 0$.

Hence,

$$p(x)dx = C_o \left[P_o(x)P_1(dx) + P_1(x)P_1(dx) + \quad \cdots \right], \quad (6.2.4)$$

where C_o is a constant of proportionality.

After using 6.1.1, we have

$$p(x)dx = C_o \; P_1(dx) \sum_{i=o}^{\infty} P_n \,(x) \quad (6.2.5)$$

or

$$p(x) = C_o \; \lambda. \quad (6.2.6)$$

To evaluate C_o, consider the integral of $p(x)$ on the interval $[0,T_s)$. The integral of this is unity, given that the call that occurred at x

was lost, since it occurs only $x < T_s$ after the start of a service.
This becomes

$$\int_o^{T_s} p(x)dx = 1 = \lambda C_o T_s \tag{6.2.7}$$

or

$$C_o = \frac{1}{\lambda T_s} \quad \text{for} \quad o \leq x < T_s . \tag{6.2.8}$$

Then p(x) becomes

$$p(x) = \lambda \frac{1}{\lambda T_s} = \frac{1}{T_s} , \text{ for } o \leq x < T_s . \tag{6.2.9}$$

We shall use this result to calculate the probability of an arrival being lost for independent servers when delayed calls are lost. This probability is denoted by P_3 and for $1-P_3$ we have,

$1-P_3$ = the probability that the server is idle when the arrival occurs plus the probability that the previous call was lost and occurred at x referenced to the arrival of the call in service, and that the next arrival occurs $y > T_s - x$ later.

This becomes, for c servers,

$$1 - P_3 = (1-P_3) \int_{T_s}^{\infty} \tau_c(y)dy + P_3 \int_{T_s}^{\infty} \int_{o}^{T_s} p(x)\ \tau_c(y-x)dxdy \qquad (6.2.10)$$

by noting that in steady state conditions P_3 for the n^{th} arrival equals P_3 for the $n\text{-}1^{st}$ arrival. Substituting 6.1.3 in 6.2.10 for c arrivals, we have

$$1 - P_3 = (1-P_3) \int_{T_s}^{\infty} \frac{\lambda(\lambda y)^{c-1}}{c-1!} e^{-\lambda y}\ dy\ +$$

$$P_3 \int_{T_s}^{\infty} \int_{o}^{T_s} \frac{1}{T_s}\ \frac{\lambda^c (y-x)^{c-1}}{c-1!}\ e^{-\lambda(y-x)}\ dxdy \qquad (6.2.11)$$

After integration and some rearranging, we have

$$1\text{-}P_3 = \frac{\dfrac{c}{\lambda T_s} - \dfrac{e^{-\lambda T_s}}{\lambda T_s} \displaystyle\sum_{i=o}^{c-1} \sum_{j=o}^{i} \frac{(\lambda T_s)^j}{j!}}{1 + \dfrac{c}{\lambda T_s} - e^{-\lambda T_s} \displaystyle\sum_{i=o}^{c-1} \frac{(\lambda T_s)^i}{i!} - \frac{e^{-\lambda T_s}}{\lambda T_s} \sum_{i=o}^{c-1} \sum_{j=o}^{i} \frac{(\lambda T_s)^j}{j!}}$$

(6.2.12)

and finally,

$$P_3 = \frac{1 - e^{-\lambda T_s} \sum_{i=o}^{c-1} \frac{(\lambda T_s)^i}{i!}}{1 - e^{-\lambda T_s} \sum_{i=o}^{c-1} \frac{(\lambda T_s)^i}{i!} + \frac{c}{\lambda T_s} - \frac{e^{-\lambda T_s}}{\lambda T_s} \sum_{i=o}^{c-1} \sum_{j=o}^{i} \frac{(\lambda T_s)^j}{j!}} . \quad (6.2.13)$$

Equation 6.2.13 is shown graphically in Figure 3 as the continuous lines. The data points are the result of a computer simulation for this case. It is interesting to note that for c = 1, 6.2.13 reduces to

$$P_3 = \frac{\lambda T_s}{1 + \lambda T_s} . \quad (6.2.14)$$

If equation 6.2.14 is considered for small values of λT_s, we have

$$P_3 \doteq \lambda T_3, \text{ for } \lambda T_s << 1 . \quad (6.2.15)$$

Comparing 6.1.7 and 6.2.15, it can be observed that we obtain an order of 2 advantage in 6.2.15. This is because in 6.1.7 signal overlapping causes loss from both signals, while in 6.2.15 the signal in service is not affected by subsequent arrivals and only arrivals that occur during a service are lost.

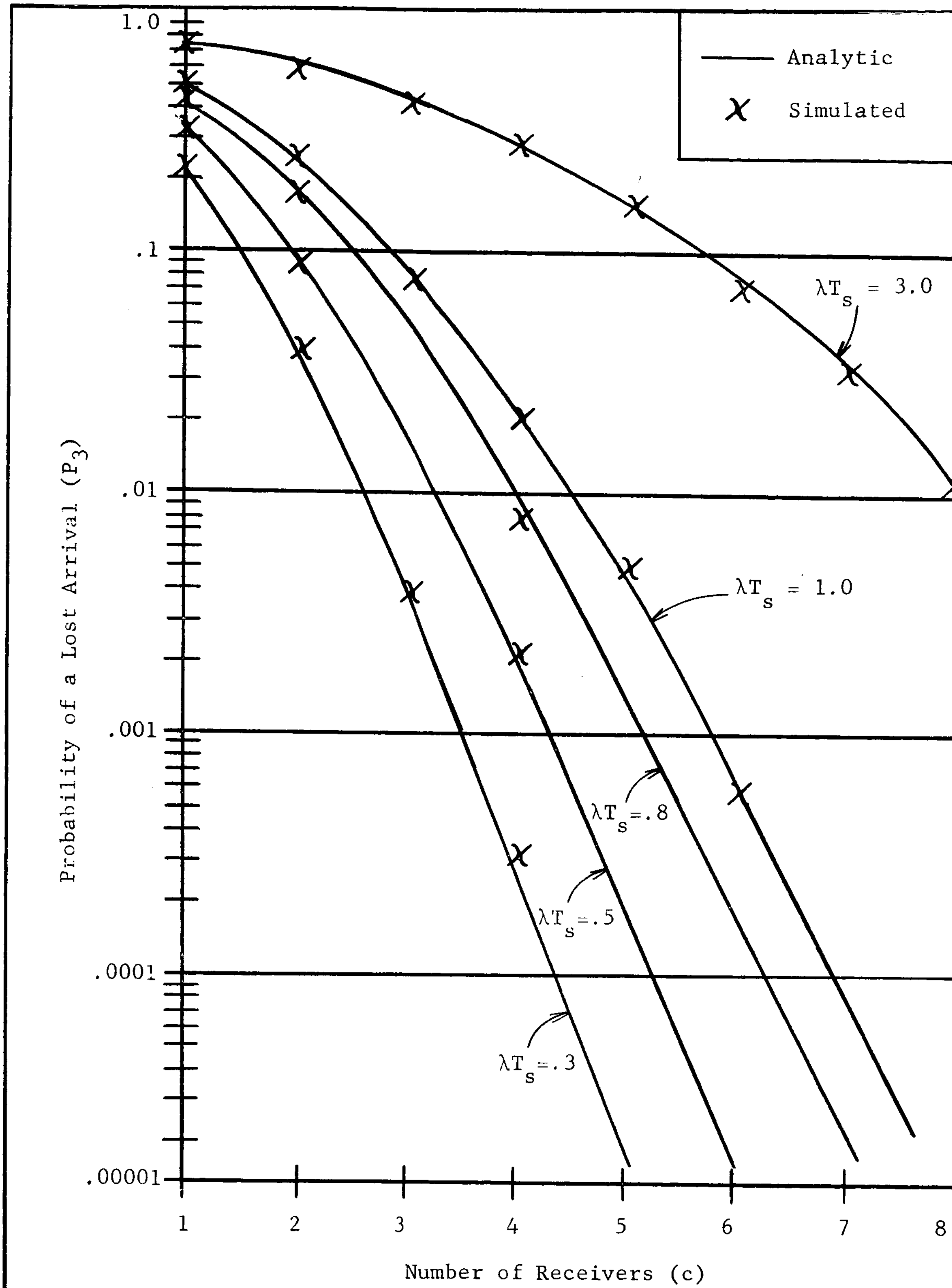

Figure 3. Probability of a Lost Arrival for Independent Receivers when Delayed Arrivals are Lost.

Case 3. Dependent receivers where delayed calls are lost—In the third case considered here, new arrivals are assigned to any receiver that is free. If none is free upon the arrival of any signal, that signal is ignored by the service system and, hence, lost information results. This type of operation has an advantage over Case 1 because in Case 3 all calls that can not be serviced completely are ignored and do not occupy server time. Case 3 has an advantage over Case 2 because, for the previous case, signals are assigned in rotation to receivers so that, for c receivers, every c^{th} arrival is assigned to the same receiver if it is free; otherwise, it is lost. In Case 3, a new arrival may be assigned to any free receiver and is not restricted by the assignment in rotation. This will enable some arrivals to be serviced that would be lost in Case 2. For the dependent server operation, a computer simulation was performed by the use of Monte Carlo techniques. The results are shown in Figure 4. It is easily seen that there is only a small difference in magnitude between the dependent situation and Case 2. Equation 6.2.13 may be used as an approximation and upper limit for Case 3.

A functional diagram of the Monte Carlo simulation procedure is shown in Figure 5. The net result is the number of serviced arrivals, the number of lost arrivals, and the total number of arrivals (10,000 in our case). The probability of an arrival being lost is calculated on a per arrival basis by finding the ratio of lost to total arrivals for each case of λT_s and c. The sample size is sufficiently large that the sample variance is reasonably small unless the total number of lost arrivals is small. This occurrence naturally corresponds to low

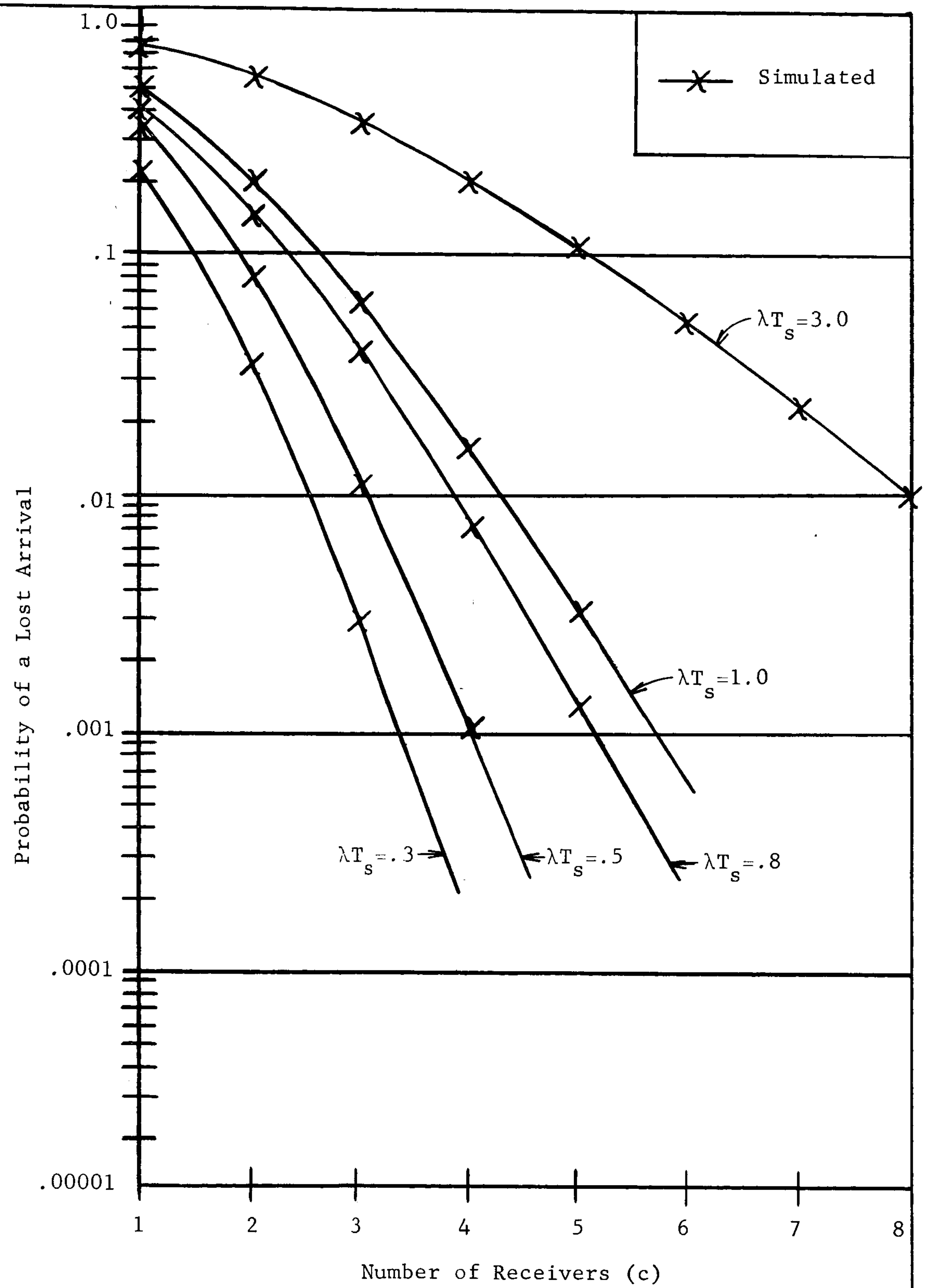

Figure 4. Probability of a Lost Arrival for Dependent Receivers when Delayed Arrivals are Lost.

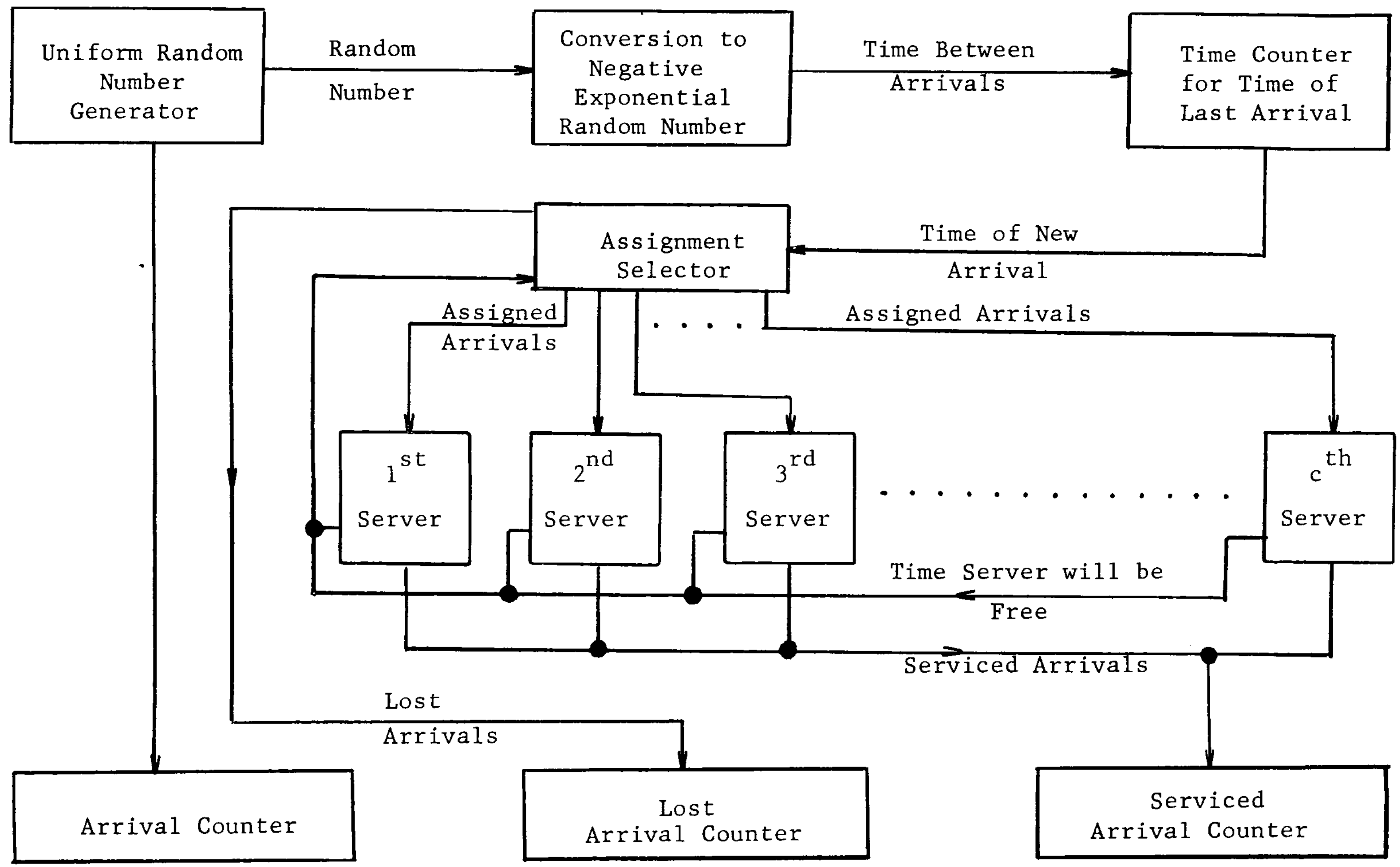

Figure 5. Functional Diagram of Monte Carlo Simulation

probabilities of loss. This can be recognized from the analytic and simulated data shown in Figure 3. The program was written in Daft Programming language compatable with the IBM 7074 available at The Pennsylvania State University. Daft is very similar to the more common Fortran Programming language. This program is shown in Appendix B.

2. Redundant data intervals

We shall assume that instead of multiple receiver operation, only one receiver is utilized. However, each source has the capability of repeating the data interval several times when it does transmit. Upon the arrival of any signal with the receiver occupied, the signal "waits" until either the server becomes free or the signal duration expires. This type of operation enables the signal to be completely served on any data interval except the last, depending upon when the server becomes unoccupied. The signal may be incompletely serviced if service does not begin before the start of the last data interval, but does begin service before the signal expires.

Let:

y = the waiting time of the n^{th} arrival, i.e., the time between arrival and start of service of the n^{th} signal,

x = the waiting time of the $n - 1^{st}$ arrival,

K = the number of the data intervals, i.e., the length of each source transmission is KT_s,

t = time between adjacent arrivals,

$W_n'(y)$ = probability density of n^{th} arrival waiting a time y for service to begin, and

$W_{n-1}'(x)$ = probability density of $n\text{-}1^{st}$ arrival waiting a time x for service to begin.

Then,

$$y = \begin{cases} x + T_s - t & \text{if} \quad x + T_s \leq KT_s \\ KT_s - t & \text{if} \quad x + T_s \geq KT_s . \end{cases} \tag{6.2.16}$$

After rearranging, this becomes

$$t = \begin{cases} x + T_s - y & \text{for} \quad x + T_s \leq KT_s \\ KT_s - y & \text{for} \quad x + T_s \geq KT_s . \end{cases} \tag{6.2.17}$$

Consider, first, the probability density $W_n'(y)$, for $y \leq T_s$. For $W_n'(y)$, we have

$W_n'(y)$ = probability density of y, given $x+T_s \geq KT_s$ plus the probability density of y, given $x+T_s \leq KT_s$ where $x > o$ plus the probability density that $y < T_s$, given that $x = o$.

This becomes

$$W_n'(y) = \int_{(K-1)T_s}^{KT_s} W_{n-1}'(x)\ \tau_1(KT_s-y)dy + \int_{o}^{(K-1)T_s} W_{n-1}'(x)\ \tau_1(x+T_s-y)dx$$

$$+ W_{n-1}(o)\ \tau_1(T_s-y) \quad \text{for} \quad y \leq T_s. \tag{6.2.18}$$

Likewise, $W_n'(y)$ for $T_s \leq y \leq KT_s$, may be described by

$W_n'(y)$ = probability density of y, given $x+T_s \geq KT_s$ plus the probability density of y, given $x+T_s \leq KT_s$.

This becomes

$$W_n'(y) = \int_{(K-1)T_s}^{KT_s} W_{n-1}'(x)\ \tau_1(KT_s-y)dx + \int_{y-T_s}^{(K-1)T_s} W_{n-1}'(x)\ \tau_1(x+T_s-y)dx. \tag{6.2.19}$$

If we concern ourselves with the steady state stochastic processes where the mean number of arrivals is equal to the mean number of services started, including partial and complete services, then,

$$W_{n-1}'(\epsilon) \longrightarrow W_n'(\epsilon) \longrightarrow W'(\epsilon)$$

and

$$W_{n-1}(0) \rightarrow W_o \ .$$

Using this notation and 6.1.3 for n = 1, 6.2.18 and 6.2.19 become

$$W'(y) = \int_{(K-1)T_s}^{KT_s} W'(x)\lambda e^{-\lambda(KT_s-y)} dx + \int_{0}^{(K-1)T_s} W'(x)\lambda e^{-\lambda(x+T_s-y)} dx$$

$$+ \ W_o \lambda e^{-\lambda(T_s-y)} \tag{6.2.20}$$

for $y \leq T_s$, and

$$W'(y) = \int_{(K-1)T_s}^{KT_s} W'(x)\lambda e^{-\lambda(KT_s-y)} dx + \int_{y-T_s}^{(K-1)T_s} W'(x)\lambda e^{-\lambda(x+T_s-y)} dx \tag{6.2.21}$$

for $T_s \leq y \leq KT_s$.

For W_o,

$$W_o = \int_{-\infty}^{0} W'(y)dy. \tag{6.2.22}$$

Let

$$1 - D_o = \int_{(K-1)T_s}^{KT_s} W'(x)dx = W(KT_s) - W\left[(K-1)T_s\right],$$

then 6.2.20 becomes

$$W'(y) = \lambda e^{-\lambda y}\left[e^{-\lambda KT_s}(1-D_o) + \int_{T_s}^{KT_s} W'(x-T_s)e^{-\lambda x}dx + W_o e^{-\lambda T_s}\right] \text{ for } y \leq T_s$$

(6.2.23)

and 6.2.22 becomes

$$W_o = \int_{-\infty}^{o} W'(y)dy = (1-D_o)e^{-\lambda KT_s} + \int_{T_s}^{KT_s} W'(x-T_s)e^{-\lambda x}dx + W_o\, e^{-\lambda T_s}.$$

(6.2.24)

Therefore,

$$W'(y) = \lambda e^{-\lambda y}\, W_o \quad \text{for} \quad y \leq T_s. \qquad (6.2.25)$$

Solving for W_o in 6.2.24, we have

$$W_o = \frac{(1-D_o)e^{-\lambda KT_s} + \int_{T_s}^{KT_s} W'(x-T_s)e^{-\lambda x}dx}{1 - e^{-\lambda T_s}}, \tag{6.2.26}$$

For $o \leq y \leq \epsilon \leq T_s$,

$$W(\epsilon) = \int_{-\infty}^{\epsilon} W'(y)dy = \int_{-\infty}^{\epsilon} \lambda e^{-\lambda y} W_o dy = W_o e^{\lambda \epsilon} \tag{6.2.27}$$

and $W'(\epsilon) = \lambda W(\epsilon)$. (6.2.28)

For $T_s \leq y \leq \epsilon \leq KT_s$, let $z = x + T_s$

in the last term of 6.2.21. Then 6.2.21 becomes

$$W'(y) = \lambda e^{\lambda y}\left[(1-D_o)e^{-\lambda KT_s} + \int_{y}^{KT_s} W'(z-T_s)e^{-\lambda z}dz\right]. \tag{6.2.29}$$

Then,

$$W(\epsilon) = \int_{-\infty}^{\epsilon} W'(y)dy = W(T_s) + \int_{T_s}^{\epsilon} W'(y)dy. \tag{6.2.30}$$

This can be shown to reduce to

$$W(\epsilon) = e^{\lambda\epsilon}\left[(1-D_o)e^{-\lambda KT_s} + \int_{\epsilon}^{KT_s} W'(x-T_s)e^{-\lambda x}dx\right] + W(\epsilon-T_s) \tag{6.2.31}$$

or

$$W(\epsilon) = \frac{W'(\epsilon)}{\lambda} + W(\epsilon-T_s). \tag{6.2.32}$$

Then, for $o \leq \epsilon \leq KT_s$, combining 6.2.28 and 6.2.32, we have

$$W(\epsilon) = \frac{W'(\epsilon)}{\lambda} + W(\epsilon-T_s)\; u(\epsilon-T_s), \tag{6.2.33}$$

where $u(\epsilon-T_s)$ is the unit step function such that

$$u(\epsilon-T_s) = \begin{cases} 1, & \epsilon \geq T_s \\ 0, & \epsilon < T_s \end{cases} .$$

Now to solve 6.2.33, we may resort to Laplace Transforms.

Let

$$\varphi(s) = \int_{o}^{\infty} e^{-s\epsilon} W(\epsilon)d\epsilon. \tag{6.2.34}$$

Taking the transform of both sides of 6.2.33, we obtain

$$\varphi(s) = \frac{s\varphi(s) - W_o}{\lambda} + e^{-sT_s} \varphi(s) \tag{6.2.35}$$

or

$$\frac{\varphi(s)}{W_o} = \frac{1}{s-\lambda + \lambda e^{-\lambda T_s}} . \tag{6.2.36}$$

This can be shown to converge for $s_r > \lambda$ if $s = s_r + j\, s_i$,

where s_r and s_i are the real and imaginary parts of s respectively.

Then

$$\frac{\varphi(s)}{W_o} = \sum_{i=o}^{\infty} \frac{(-\lambda)^i e^{-isT_s}}{(s-\lambda)^{i+1}} . \tag{6.2.37}$$

Taking the inverse transform of 6.2.37, we have

$$\frac{W(\epsilon)}{W_o} = \sum_{i=o}^{\infty} \frac{(-\lambda)^i \, (\epsilon - T_s)^i \, e^{-\lambda(\epsilon - iT_s)} \, u(\epsilon - iT_s)}{i!} \quad . \tag{6.2.38}$$

Let $\left\lfloor \frac{\epsilon}{T_s} \right\rfloor$ represent the largest integer such that $\left\lfloor \frac{\epsilon}{T_s} \right\rfloor \leq \frac{\epsilon}{T_s}$.

Then

$$W(\epsilon) = W_o \sum_{i=o}^{\left\lfloor \frac{\epsilon}{T_s} \right\rfloor - 1} \frac{(-\lambda)^i \, (\epsilon - iT_s)^i e^{\lambda(\epsilon - iT_s)}}{i!} \quad . \tag{6.2.39}$$

Also,

$$W(KT_s) = 1 = W_o \sum_{i=o}^{K-1} \frac{\left[(-\lambda T_s)(K-i)\right]^i \, e^{\lambda T_s (K-i)}}{i!} \tag{6.2.40}$$

and

$$W_o = \frac{1}{\sum\limits_{i=o}^{K-1} \frac{\left[(-\lambda T_s)(K-i)\right]^i \, e^{\lambda T_s (K-i)}}{i!}} \quad . \tag{6.2.41}$$

Then from 6.2.39 and 6.2.41, we may find

$$D_o = W\left[(K-1)T_s\right] = W_o \sum_{i=o}^{K-2} \frac{\left[(-\lambda T_s)(K-1-i)\right]^i}{i!} e^{\lambda T_s (K-1-i)}$$

(6.2.42)

or

$$W\left[(K-1)T_s\right] = \frac{\displaystyle\sum_{i=o}^{K-2} \frac{\left[(-\lambda T_s)(K-1-i)\right]^i e^{\lambda T_s(K-1-i)}}{i!}}{\displaystyle\sum_{i=o}^{K-1} \frac{\left[(-\lambda T_s)(K-i)\right]^i e^{\lambda T_s(K-i)}}{i!}} .$$

(6.2.43)

Since $W\left[(K-1)T_s\right]$ is the probability that an arrival will wait $(K-1)T_s$ or less, $1-W\left[(K-1)T_s\right]$ is the probability that an arrival must wait more than $(K-1)T_s$ for service, and because the signal duration is KT_s, this implies that only an incomplete service is possible. Therefore,

$$P_4 = 1-W\left[(K-1)T_s\right] . \qquad (6.2.44)$$

Figure 6 shows analytical curves of the probability of an incompletely serviced arrival for specific values of λT_s versus the number of data or information intervals. The data points on the curves are the results of a computer simulation of the servicing operation.

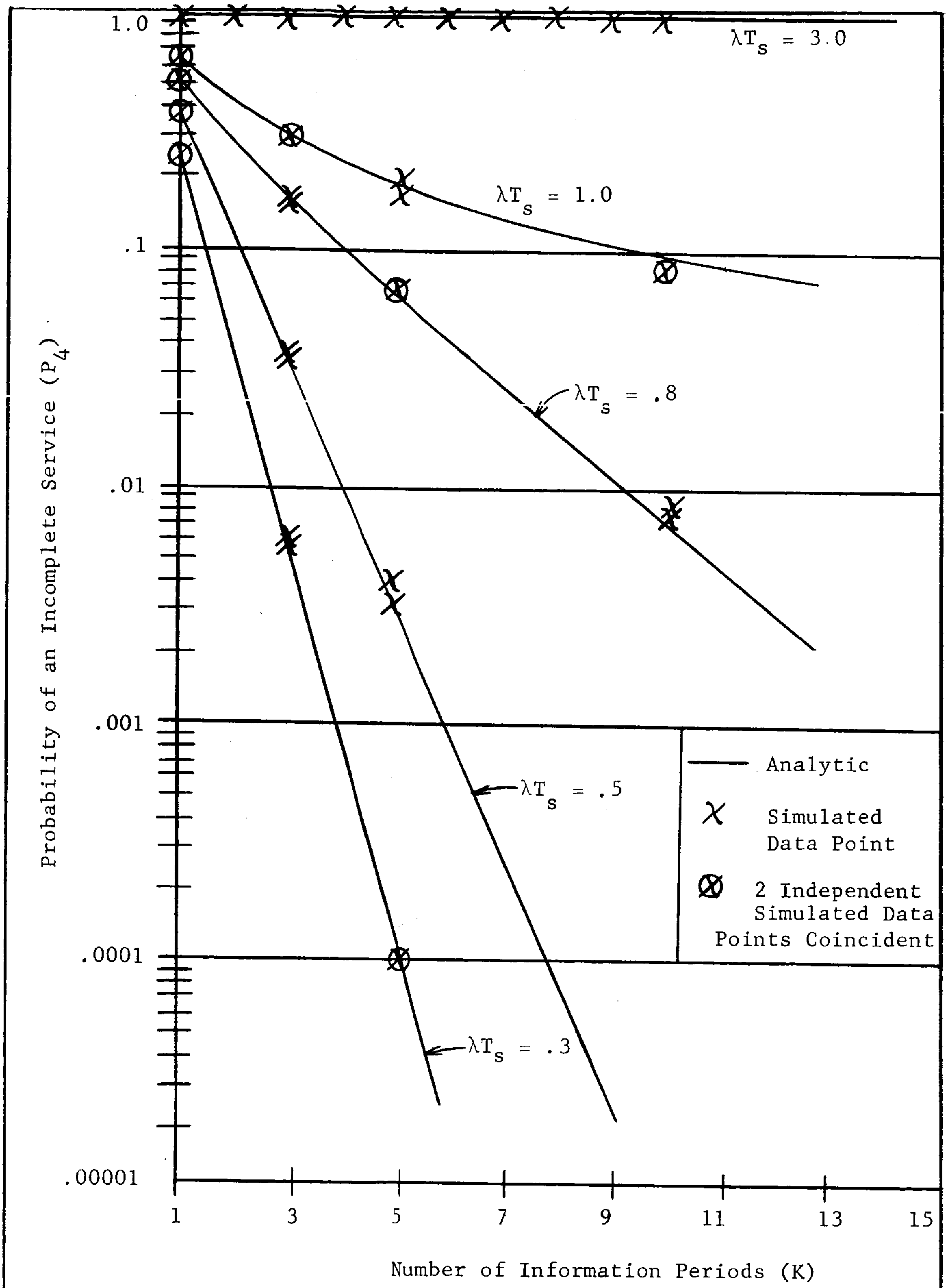

Figure 6. Probability of an Incomplete Service Versus the Number of Information Periods.

VII. PROBABILITY OF INFORMATION LOSS, II

Misinterpreting the Symbols

In this chapter we shall consider the information loss due to an erroneous decision concerning which symbol was actually transmitted when the received signal is corrupted by noise and fading in the communications channel.

The probability of symbol error is necessary for a complete calculation of total information loss probability. The results of this chapter and the previous chapter will be combined in Chapter VIII to provide the total information loss probability for the whole system.

Lindsay (1964) has considered the general case of error probabilities in an N state system with multichannel reception. His result includes, as a special case, the single channel problem of interest here. However, the derivation is very complex and several intermediate steps which provide a more complete understanding of the decisional and error processes are not readily available in published literature. It was, therefore, desirable to develop independently a unified and coherent treatment of the results needed in this study, starting with the simpler assumption of single channel. It was considered that this treatment allows a more complete understanding of the decision process on which the final error rate was based. The special case where we have a single channel and binary state symbols can also be found from Lindsay's more general N state expression, and was also derived previously in Turin (1958).

Assume a frequency shift keying system in which the symbols are identified by the transmission of discrete frequencies ω_i for a period T, such that $\frac{2\pi}{\omega_i T}$ are unique integers. The symbols are then represented by the time function

$$x_i(t) = S_o \cos \omega_i t, \text{ for } mT < t < (m+1)T,$$

where m is an arbitrary integer, and

$$\frac{1}{E} \int_{mT}^{(m+1)T} x_i(t)\, x_j(t) dt = \delta_{ij},$$

where $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ij} = 1$ if $i = j$, and E is the transmitter energy for each symbol. The set of frequencies ω_i will be denoted by Ω_o. When the i^{th} symbol state is transmitted at $(t - \tau_p)$, the signal y (t) is received at a time equal to the propagation time (τ_p) later. Let A_o/S_o be the average ratio of received to transmitted amplitude for free space conditions, for the duration of the symbol (T). In terms of the receiver time,

$$y(t) = a\, A_o \left[\cos\left(\omega_i t + \theta \right) \right] + n(t) ,$$

where (a) is the magnitude of the fading variable, θ is the random phase shift, and n(t) is assumed to be additive white Gaussian noise.

The fading factor (a) is assumed to be slowly varying so that (a) may be assumed constant for a symbol duration.

7.1. Fading Phenomenon

The fading factor (a) may be recognized to be a random variable in most applications where atmospheric propagation is utilized. For generality, a fading relationship may be chosen consisting of a constant fading term and a random component. This is illustrated in Figure 7. y(t) may be represented by

$$y(t) = \mathrm{Re}\left[A_o\left(\alpha e^{-j\delta} + be^{-j\varphi}\right)e^{j\omega_i t}\right] + n(t). \tag{7.1.1}$$

In this context $\alpha e^{-j\delta}$ is a fixed or specular component and $be^{-j\varphi}$ is a random or scatter component.

Assume that b has a Rayleigh distribution and φ obeys the uniform distribution, for an interval $(0,2\pi)$, and b and φ are independent. Then

$$p(b,\varphi) = p(b)\ p(\varphi) \tag{7.1.2}$$

and

$$p(b,\varphi) = \frac{b}{2\pi\sigma_b^2}\ e^{-b^2/2\sigma_b^2}\ . \tag{7.1.3}$$

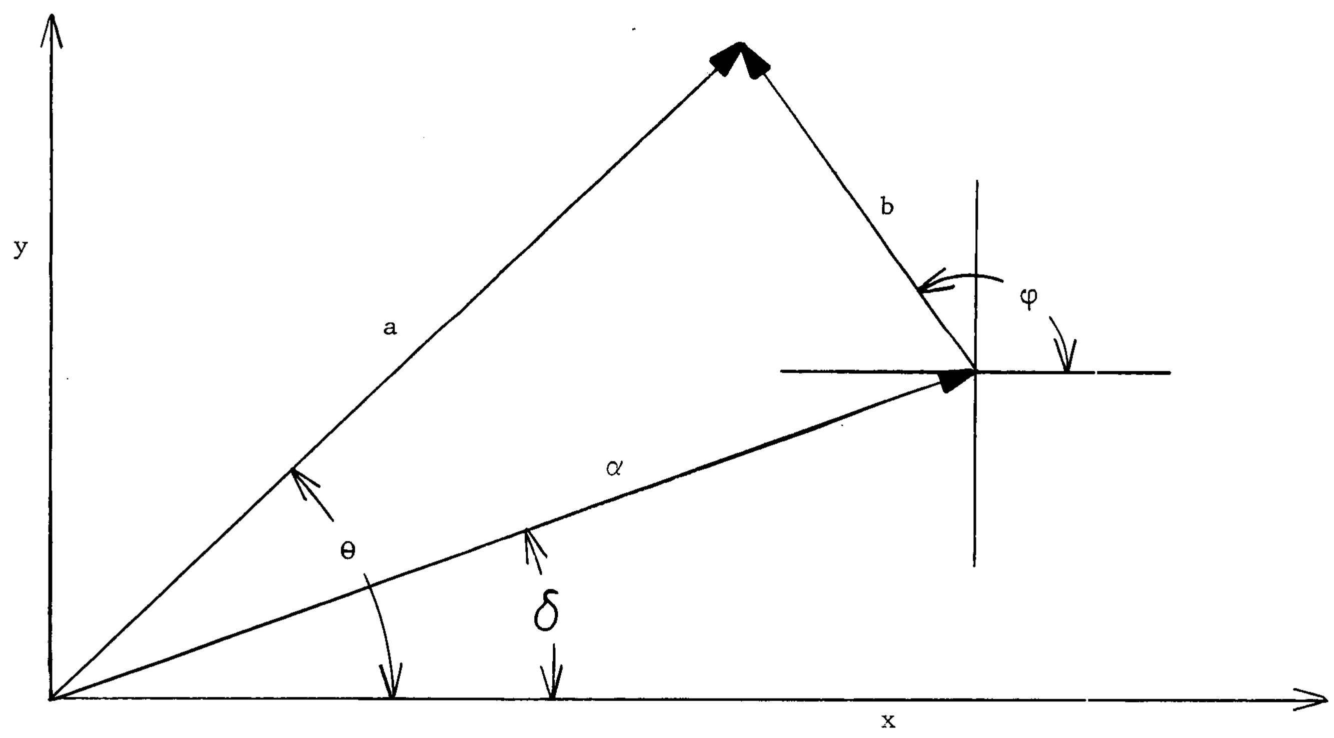

Figure 7. Vector Geometry of the Fading Variable

Since

$$a^2 = x^2 + y^2, \tag{7.1.4}$$

$$x = b \cos \varphi + \alpha \cos \delta \ , \text{ and} \tag{7.1.5}$$

$$y = b \sin \varphi + \alpha \sin \delta \ , \tag{7.1.6}$$

then

$$a^2 = b^2 + \alpha^2 + 2b\alpha \cos (\varphi - \delta). \tag{7.1.7}$$

But also from the Law of Cosines

$$b^2 = a^2 + \alpha^2 - 2a\alpha \cos (\varphi - \delta). \tag{7.1.8}$$

Changing variables,

$$p(a,\theta) = p\left[b(a,\theta),\ \varphi(a,\theta)\right] \left|J(b,\varphi;a,\theta)\right| , \tag{7.1.9}$$

where $|J|$ is the absolute value of the Jacobian. From 7.1.8 we have

$$\frac{\partial b}{\partial a} = \frac{a - \alpha \cos(\theta - \delta)}{b} \tag{7.1.10}$$

and

$$\frac{\partial b}{\partial \theta} = a\alpha \sin (\theta - \delta). \tag{7.1.11}$$

From 7.1.7 we have

$$\frac{\partial\varphi}{\partial a} = \left[\frac{b+\alpha\cos(\theta-\delta)}{b\alpha\sin(\varphi-\delta)}\right]\frac{\partial b}{\partial a} - \frac{a}{b\alpha\sin(\varphi-\delta)} \tag{7.1.12}$$

and

$$\frac{\partial\varphi}{\partial\theta} = \left[\frac{b+\alpha\cos(\varphi-\delta)}{b\alpha\sin(\varphi-\delta)}\right]\frac{\partial b}{\partial\theta}\,. \tag{7.1.13}$$

Then from 7.1.10, 7.1.11, 7.1.12, and 7.1.13

$$\left|J\,(b,\varphi;\ a,0)\right| = \frac{a}{b}\,.$$

Therefore, the probability density for the fading factor in polar coordinates is

$$p(a,\theta) = \frac{a}{2\pi\sigma_b^2}\, e^{-\frac{1}{2\sigma_b^2}\left[a^2+\alpha^2-2a\alpha\cos(\theta-\delta)\right]}\,. \tag{7.1.14}$$

This is the well known Rician probability density function.

This particular density function has an advantage over the more common assumption of Rayleigh fading. It incorporates a constant fading component as well as the Rayleigh term, and, hence, it can be manipulated to provide a constant fading term, mixed fading, Rayleigh fading, or an approximate Gaussian fading.

7.2. Decision Rules

We require the probability that $x_i(t)$ was transmitted, given that $y(t)$ was received. Using the Bayes equality formulation, this is

$$P\left[x_i(t)/y(t)\right] = \frac{p\left[y(t)/x_i(t)\right]}{p\left[(y(t)\right]} P\left[x_i(t)\right].$$

If we assume that each $x_i(t)$ is equally likely, we have for $i = 1,2,\ldots n_s$, where n_s is the number of states for each symbol,

$$P\left[x_i(t)\right] = \frac{1}{n_s} .$$

Since $P\left[x_i(t)\right]$ is known and $p\left[y(t)\right]$ is fixed for a given receiver, the problem of computing $P\left[x_i(t)/y(t)\right]$ is just the problem of computing the likelihoods $\Lambda_i = p\left[y(t)/x_i(t)\right]$. Also, the noise $n(t)$ is just

$$n(t) = y(t) - aA_o \cos\left[\omega_i t + \theta\right] . \qquad (7.2.1)$$

Therefore, the likelihoods Λ_i are just the probability densities that the noise waveform is given by 7.2.1 for each possible value of i. Using Woodward's formulation, given in Woodward (1953), for p $\left[n(t)\right]$, we have

$$p\left[n(t)\right] = K_o\, e^{-\frac{1}{N_o}\int_0^T n^2(t)dt} = \Lambda_i, \qquad (7.2.2)$$

where N_o is the noise power density and K_o is a constant of proportionality. Using 7.2.1, p $\left[n(t)\right]$ becomes

$$p(y/x_i,a,\theta) = K_o\, e^{-\frac{1}{N_o}\int_0^T \left[y(t) - aA_o \cos(\omega_i t + \theta)\right]^2 dt} \qquad (7.2.3)$$

or

$$p(y/x_i,a,\theta) = K_o\, e^{-\frac{1}{N_o}\int_0^T \left[y^2(t)+a^2A_o^2 \cos^2(\omega_i t+\theta)-2y(t)aA_o\cos(\omega_i t+\theta)\right]dt}. \qquad (7.2.4)$$

Let

$$B(y) = K_o \, e^{-\frac{1}{N_o} \int_0^T y^2(t)dt} \tag{7.2.5}$$

in 7.2.3 and the free space received signal to noise ratio (R) be

$$R = \frac{1}{N_o} \int_0^T A_o^2 \cos^2 (\omega_i t + \theta) dt. \tag{7.2.6}$$

Then 7.2.3 becomes

$$p(y/x_i, a, \theta) = B(y) \, e^{-a^2 R + \frac{2aA_o}{N_o} \int_0^T y(t) \cos (\omega_i t + \theta) dt} . \tag{7.2.7}$$

Using the expansion

$$\cos (\omega_i t + \theta) = \cos \omega_i t \cos \theta - \sin \omega_i t \sin \theta \tag{7.2.8}$$

in 7.2.7, we have

$$p(y/x_i,a,\theta) = B(y)e^{-a^2R + \frac{2aA_o}{N_o}\int_o^T y(t)\left[\cos\omega_i t\cos\theta - \sin\omega_i t\sin\theta\right]dt} \quad . \tag{7.2.9}$$

Let

$$X_i = \int_o^T A_o y(t)\cos\omega_i t\, dt \tag{7.2.10}$$

and

$$Y_i = \int_o^T A_o y(t)\sin\omega_i t\, dt \;. \tag{7.2.11}$$

Then 7.2.9 becomes

$$p(y/X_i,Y_i,a,\theta) = B(y)e^{-a^2R + \frac{2a}{N_o}(X_i\cos\theta - Y_i\sin\theta)} \quad . \tag{7.2.12}$$

Now let

$$Z_i^2 = X_i^2 + Y_i^2 \tag{7.2.13}$$

and

$$\varphi_i = \tan^{-1} \frac{Y_i}{X_i}, \text{ for } o \leq \varphi_i \leq 2\pi. \tag{7.2.14}$$

Then,

$$p(y/Z_i, a, \theta, \varphi_i) = B(y)e^{-a^2 R + \frac{2a}{N_o}\left[Z_i \cos(\varphi_i + \theta)\right]}. \tag{7.2.15}$$

Using 7.1.15 and the fact that

$$p(y/Z_i, \varphi_i) = \int_o^{\infty} \int_o^{2\pi} p(a,\theta)\, p(y/Z_i, a, \theta, \varphi_i) da d\theta, \tag{7.2.16}$$

7.2.15 becomes

$$p(y/Z_i, \varphi_i) = \frac{B(y)}{1+g\rho}\, e^{\frac{\alpha^2}{-2\sigma_b^2} + D_i^2\ \alpha/2\ (1+g\rho)}, \tag{7.2.17}$$

where

$$D_i = \frac{4Z_i^2 A_o}{N_o^2} + \frac{\alpha^2}{\sigma_b^4} + \frac{4Z_i \alpha}{N_o \sigma_b^2} \cos(\varphi_i + \delta). \tag{7.2.18}$$

Also define the ratio (g) of the received energy from the random component to the received energy from the fixed component as

$$g = \frac{2\sigma_b^2 R}{\alpha^2 R} = \frac{2\sigma_b^2}{\alpha^2}, \tag{7.2.19}$$

and the energy of the fixed channel is

$$\rho = \alpha^2 R. \tag{7.2.20}$$

It is necessary to note that

$$I_o(x) = \frac{1}{2\pi} \int_o^{2\pi} e^{x \cos(\varphi_i + \theta)} d\theta \tag{7.2.21}$$

is the modified Bessel function and that

$$\int_0^{\infty} x\, e^{-a^2x^2}\, I_o(bx)dx = \frac{e^{b^2/4a^2}}{2a^2} . \qquad (7.2.22)$$

Similarly,

$$p(y/Z_i) = \int_0^{2\pi} p(y/Z_i,\varphi_i)\; p(\varphi_i)\; d\varphi_i \; . \qquad (7.2.23)$$

Using this and 7.2.17 and for $p(\varphi_i) = \frac{1}{2\pi}$, we have for noncoherent reception,

$$p(y/Z_i) = \frac{B(y)}{1+g\rho}\, I_o\left[\frac{2Z_i\ \alpha\sigma_b^2}{1+g\rho}\right] e^{-\left\{\frac{\left[\alpha^2\ (1+g\rho)\right]+\left[\frac{4Z_i^2}{N_o^2}+\frac{\alpha^2}{\sigma_b^4}\right]}{2(1+g\rho)}\right\}\sigma_b^2} . \qquad (7.2.24)$$

Equation 7.2.24 is the equation of the probability density of y(t), given Z_i, where Z_i^2 is the sum of the squares of the integrals of the signals formed by multiplying y(t) by cos $\omega_i t$ and sin $\omega_i t$, respectively. Z_i is precisely the envelope of the cross correlation of y(t) and the i^{th} stored frequency. This type of receiver is shown in Figure 8. Since there is a receiver similar to Figure 8 for every symbol state (n_s), or since frequency shift keying is used, there is

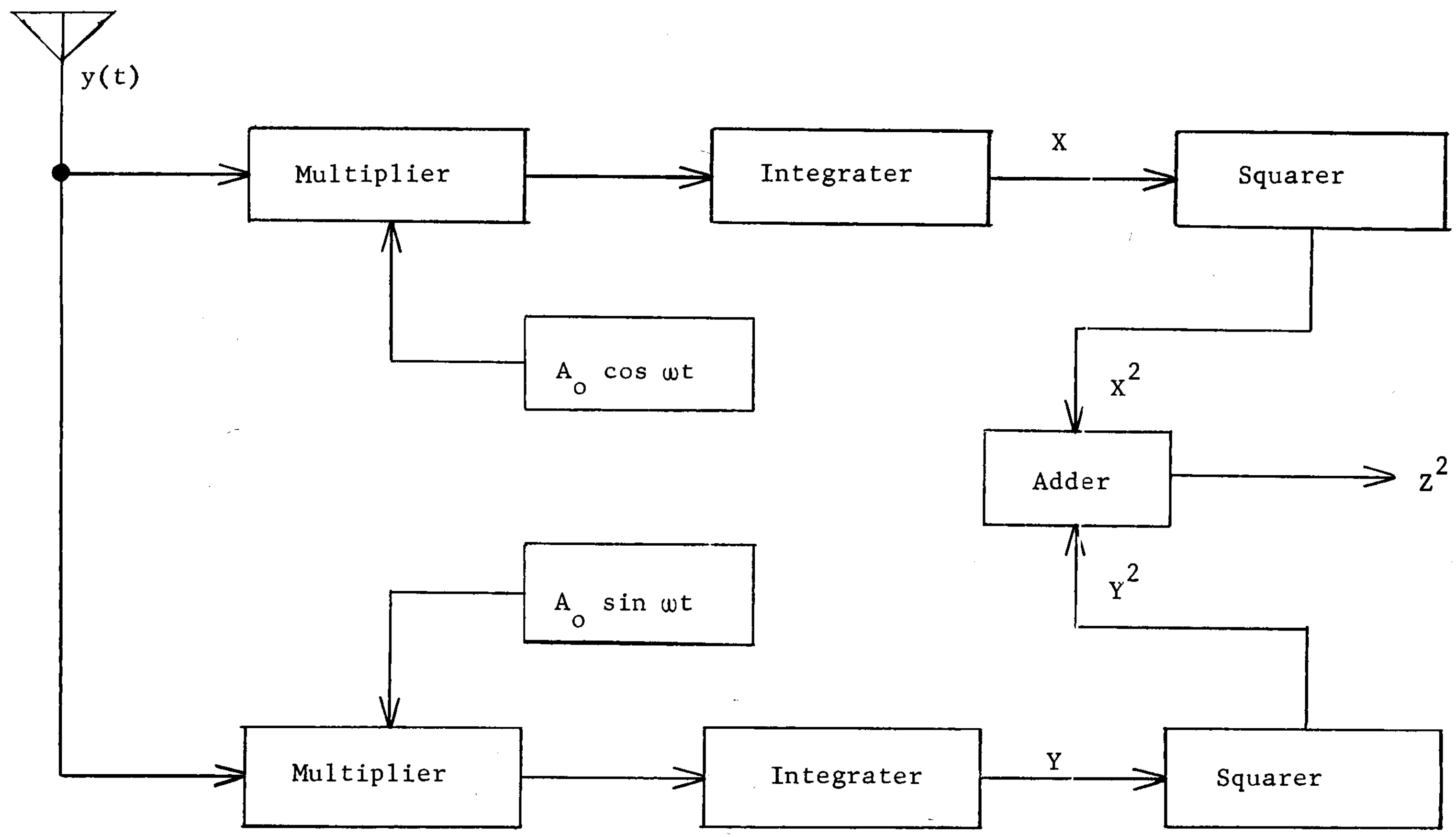

Figure 8. Functional Diagram of the Receiver

one for every frequency of the set Ω_o. The decision as to which symbol state was transmitted is accomplished by choosing the maximum value of Z_j where $j = 1,2 \ldots. n_s$. The ω_j associated with the maximum Z_j is denoted by $\widehat{\omega_j}$, and $\widehat{Z_j}$ implies $\widehat{\omega_j}$. The value of each Z_j is sampled at $t = T$ and the maximum selected. This type of reception may be accomplished alternately by the use of matched filters, such that each filter impulse response is given by

$$h_j(\tau) = A_o \cos\left[\omega_j(T-\tau)\right] . \qquad (7.2.25)$$

The envelope of each filter output is similarly sampled at the end of each symbol interval, and the largest output is selected to imply the state which was transmitted.

The selection of the largest envelope sample is equivalent to selecting the largest likelihood

$$\widehat{\Lambda}_i = p(y/\widehat{Zi}) . \qquad (7.2.26)$$

This type of posterior computer minimizes the error probability when the a priori probabilities of each symbol state are equally likely and are transmitted with equal energies.

7.3. Probability of a Symbol Error

In order to calculate the probability of making an error, we shall calculate, first, the probability of making a correct decision. To do this, it is necessary to find the probability distribution of the envelope power for the different symbol state receivers. The probability that the symbol will be correctly interpreted is just the probability that the envelope power is greatest in the receiver whose stored frequency corresponds to the symbol state frequency that was transmitted. The envelope power will be calculated assuming both signal and noise. For the n_s-1 receivers corresponding to symbols or frequencies which were not transmitted, the received power can be determined as the limiting case where the received signal component tends to zero.

For one of the receivers, the received voltage is a combination of the signal and noise components, as expressed previously in 7.2.1, and rearranged here for convenience as 7.3.1. This is given as

$$y(t) = aA_o \cos (\omega_i t + \theta) + n(t). \tag{7.3.1}$$

Now assuming

$$E\left[n(t) \right] = 0, \tag{7.3.2}$$

the expected value of X_i in 7.2.10 is

$$E(X_i) = aE_r \cos \theta \tag{7.3.3}$$

and the expected value of Y_i from 7.2.11 is

$$E(Y_i) = -a\, E_r \sin \theta, \tag{7.3.4}$$

where

$$E_r = A_o^2 \int_0^T \cos^2 (\omega_i t + \theta)\, dt. \tag{7.3.5}$$

If n(t) is assumed to be wideband, then the noise auto-correlation function $\varphi(\tau)$ becomes

$$\varphi(\tau) = \overline{n(t)\, n(t-\tau)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{N_o}{2} e^{i\omega\tau}\, d\omega = \frac{N_o}{2} \delta(\tau). \tag{7.3.6}$$

The variance about the mean for X_i and Y_i is

$$E(X_i^2) - \left[E(X_i)\right]^2 = \sigma_{X_i}^2 \tag{7.3.7}$$

and

$$E(Y_i^2) - \left[E(Y_i)\right]^2 = \sigma_{Y_i}^2 . \tag{7.3.8}$$

7.3.7 and 7.3.8 may be shown to be

$$\sigma_{X_i}^2 = \sigma_{Y_i}^2 = \frac{E_r N_o}{2} . \tag{7.3.9}$$

Similarly, $E(X_i Y_i)$ may be shown to be

$$E(X_i Y_i) = 0. \tag{7.3.10}$$

X_i and Y_i, given a and θ, are independent and subject to the normal or Gaussian probability density function. Then

$$p(X_i, Y_i/a, \theta) = \frac{1}{\pi E_r N_o} e^{-\left[\frac{(X_i - a E_r \cos\theta)^2 + (Y_i + a E_r \sin\theta)^2}{E_r N_o}\right]} . \tag{7.3.11}$$

Let

$$M_i = \frac{X_i^2 + Y_i^2}{E_r N_o} \tag{7.3.12}$$

and

$$\varphi_i = \tan^{-1} \frac{Y_i}{X_i} . \tag{7.3.13}$$

M_i may be recognized as a normalized envelope to noise ratio. Now, changing the variables, 7.3.11 becomes

$$p(M_i,\varphi_i/a,\theta) = \left|J(X_i,Y_i;M_i,\varphi_i)\right| \frac{e^{-M_i-a^2R + 2a\sqrt{M_iR}\cos(\theta+\varphi_i)}}{\pi E_rN_o}, \tag{7.3.14}$$

where

$$\left|J(X_i,Y_i;M_i,\varphi_i)\right| = \frac{E_rN_o}{2}. \tag{7.3.15}$$

Then 7.3.14 becomes

$$p(M_i,\varphi_i/a,\theta) = \frac{1}{2\pi} e^{-M_i -a^2R+2a\sqrt{M_iR}\cos(\theta+\varphi_i)}. \tag{7.3.16}$$

Now, recognizing that

$$p(M_i,\varphi_i,a,\theta) = p(M_i,\varphi_i/a,\theta)\ p(a,\theta)$$

we may substitute 7.3.16 and 7.1.14 in the above equation, and this yields

$$p(M_i,\varphi_i,a,\theta) = \frac{a}{4\pi^2\sigma_b^2}\, e^{-\left[\dfrac{M_i\ 2\sigma_b^2 + \alpha^2 + a^2(1 + 2\sigma_b^2 R)}{2\sigma_b^2} - a\,D\cos(\theta+\gamma)\right]},$$

(7.3.17)

where

$$D^2 = 4M_iR + \frac{\alpha^2}{\sigma_b^4} + 4\sqrt{M_iR}\ \frac{\alpha}{\sigma_b^2}\ \cos(\varphi_i + \delta) \qquad (7.3.18)$$

and

$$\gamma = \tan^{-1}\left[\frac{2\sqrt{M_iR}\ \sin\varphi_i - \dfrac{\alpha}{\sigma_b^2}\ \sin\delta}{2\sqrt{M_iR}\ \cos\varphi_i - \dfrac{\alpha}{\sigma_b^2}\ \cos\delta}\right]. \qquad (7.3.19)$$

Eliminating θ by integrating over its range $(0,2\pi)$ gives

$$p(M_i,\varphi_i,a) = \frac{a}{a\pi\sigma_b^2}\, e^{-\left[\dfrac{2M_i\sigma_b^2 + \alpha^2 + a^2\,(1 + 2\sigma_b^2 R)}{2\sigma_b^2}\right]} I_o(aD).$$

(7.3.20)

The variable (a) may be eliminated, similarly, by integrating over its range $(0,\infty)$, and this becomes

$$p(M_i,\varphi_i) = \frac{e^{-\left[\frac{(2M_i\sigma_b^2+\alpha^2)}{2\sigma_b^2} + \frac{4RM_i\,\sigma_b^2+\left(\alpha^2/\sigma_b^2\right)+4\alpha\sqrt{M_iR}\,\cos\,(\varphi_i+\delta)}{2(1+2\sigma_b^2R)}\right]}}{2\pi(1+2\sigma_b^2R)} . \tag{7.3.21}$$

Now, integrating φ_i over the range $(0,2\pi)$ yields

$$p(M_i) = \frac{e^{-\left[\frac{M_i+\alpha^2R}{1+2\sigma_b^2R}\right]}\,I_o\left[2\alpha\sqrt{M_iR}/(1+\sigma_b^2R)\right]}{1+2\sigma_b^2R} . \tag{7.3.22}$$

The physical meaning of the terms of 7.3.22 may be more readily seen by recalling from 7.2.19 and 7.2.20 that the received ratio (g) of the random component to the fixed component is

$$g = 2\sigma_b^2/\alpha^2 \tag{7.3.23}$$

and the received signal to noise ratio in the fixed channel (ρ) is

$$\rho = \alpha^2 R. \tag{7.3.24}$$

Using 7.3.23 and 7.3.24, 7.3.22 becomes

$$p(M_i) = \frac{e^{-\left[\frac{M_i+\rho}{1+g\rho}\right]} I_o\left[2\alpha\sqrt{M_i R} / (1+g\rho)\right]}{1 + g\rho} . \tag{7.3.25}$$

Equation 7.3.25 gives the general case where both signal and noise are present, and, thus, corresponds to the one receiver whose stored frequency is the same as the frequency of the transmitted symbol. Let the subscript i = h denote the receiver with signal and noise. The envelope power for the receivers where the stored frequency differs from the transmitted frequency will contain noise only. The probability density function of the envelope power may be calculated from 7.3.25 by considering the limiting case where the received signal power tends to zero. For this consideration,

$$g \longrightarrow 0,$$

$$\rho \longrightarrow 0, \text{ and}$$

$$\alpha \longrightarrow 0.$$

Then for $i \neq h$, 7.3.25 becomes

$$p(M_i) = e^{-M_i} , \qquad (7.3.26)$$

since $I_o(0) = 1$, and for $i = h$,

$$p(M_h) = \frac{e^{-\left[\frac{M_h + \rho}{1 + g\rho}\right]} I_o\left[2\alpha\sqrt{M_h R} \;/\; (1+g\rho)\right]}{1 + g\rho} . \qquad (7.3.27)$$

The probability of making a correct decision is the probability that each receiver envelope power M_i, where noise only is present, will be less than the envelope power M_h for the receiver with the signal and noise. Then for n_s possible symbols, $n_s - 1$ receivers will contain noise only, and one will contain the signal and noise. The probability of being correct is

$$1 - P_e(n_s) = \int_o^{\infty} p(M_h) \left[\int_o^{M_h} p(M_i)\, dM_i \right]^{n_s - 1} dM_h \quad . \qquad (7.3.28)$$

7.3.28 becomes, after substituting 7.3.26 and 7.3.27,

$$1-P_e(n_s) = \int_0^{\infty} p(M_h) \sum_{n=0}^{n_s-1} (-1)^n \binom{n_s-1}{n} e^{-nM_h} \, dM_h \tag{7.3.29}$$

or

$$P_e(n_s) = \sum_{n=1}^{n_s-1} \frac{(-1)^{n+1} \binom{n_s-1}{n} e^{-n\rho/(n+1+ng\rho)}}{n+1+ng\rho} . \tag{7.3.30}$$

For the special binary case where $n_s = 2$, we have

$$P_e(2) = \frac{e^{-\rho/(2+g\rho)}}{2+g\rho} . \tag{7.3.31}$$

For $g\rho << 2$, 7.3.31 reduces to

$$P_e(2) \doteq \frac{e^{-\rho/2}}{2} . \tag{7.3.32}$$

This result is shown for the graph indicated by $g \leq .01$ and occurs for ranges of $P_e(2)$ and ρ as shown. In this case, the probability of error is independent of g. This corresponds to more energy in the specular channel than in the random channel. Figure 9 shows that for larger values of g corresponding to more energy in the random channel than in the specular, the probability of error is smaller for values of $\rho < 10$. If the random to specular ratio is $< .1$, the probability of error falls off faster than for $g \geq 1$ and for $p \geq 3$. If $g\rho >> 2$, 7.3.32 becomes

$$P_e(2) \doteq \frac{e^{-1/g}}{g\rho} \quad . \tag{7.3.33}$$

If $g >> 1$, 7.3.33 becomes

$$P_e(2) \doteq \frac{1}{g\rho} \quad . \tag{7.3.34}$$

This corresponds to most of the energy being in the random channel and approaches the result for the Rayleigh fading assumption.

The results of symbol error probabilities will be considered in the next chapter, where we shall find the total system information loss. Equation 7.3.30 appears in a similar form in Lindsay (1964) as a special case of his more general expression, and 7.3.31 appears, similarly, in Turin (1958). The special case of binary signals, described by 7.3.31, is shown in Figure 9.

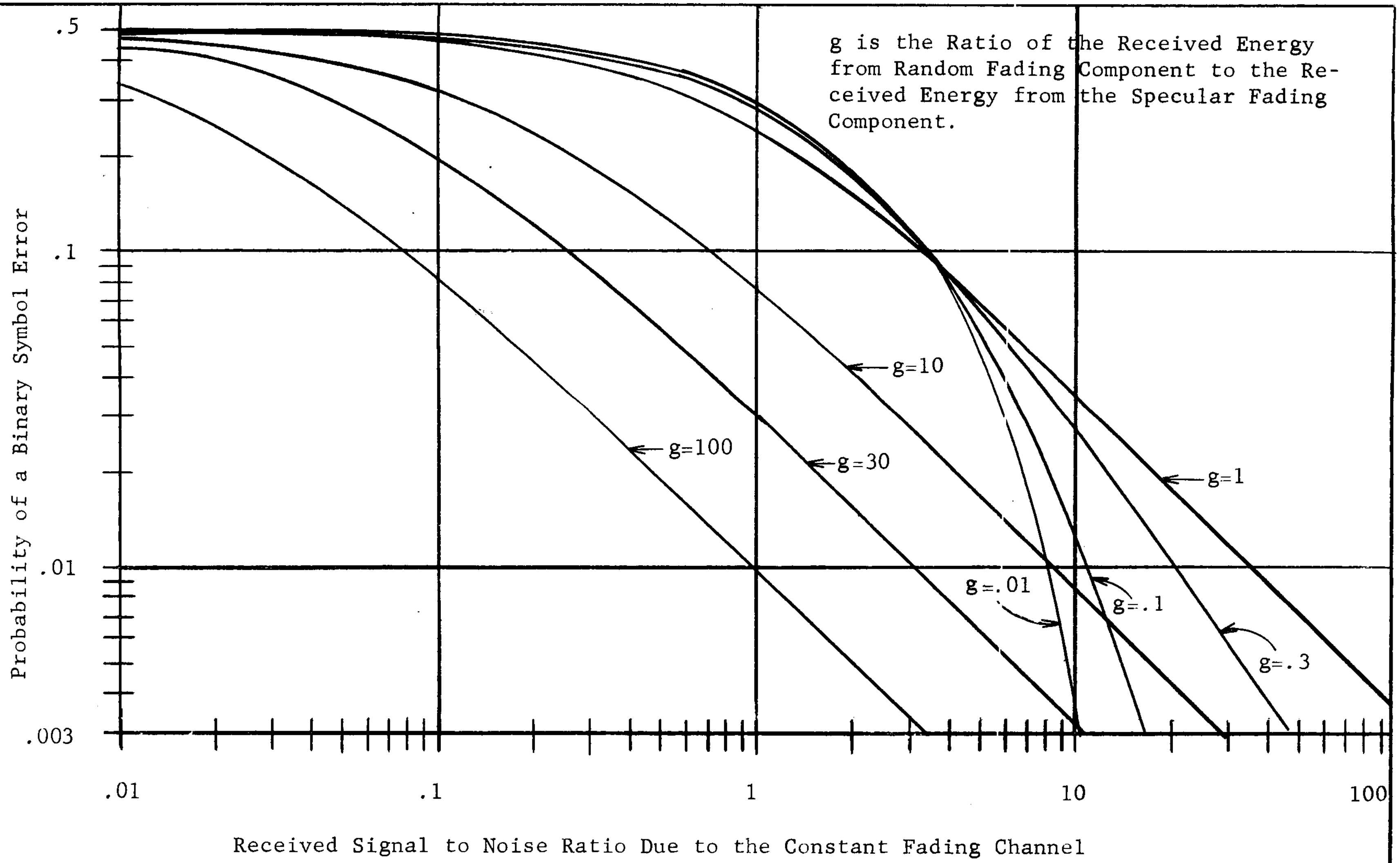

Figure 9. Probability of Binary Error Versus Received Signal to Noise Ratio Due to the Constant Fading Channel and the Energy Ratio of the Random to the Constant Channels.

VIII. RESULTS AND EXAMPLES

8.1. Total Information Loss

In Chapter VI, the results of information loss due to the servicing procedures were considered. In Chapter VII, we have considered the other contribution to information loss $P_e(n_s)$ which is the result of misinterpreting the transmitted symbol. In a message m_o symbols long, assuming independent trials for interpreting each symbol, we have

$$P_M = 1 - \prod_{j=1}^{m_o} \left[1 - P_e(n_s, j)\right] , \tag{8.1.1}$$

where P_M is the probability of an error in a message m_o symbols long, and $P_e(n_s, j)$ the probability of an error in the j^{th} symbol.

The probability of the total information loss for the system, then, is given by

$$P_T = 1 - (1-P_I)(1-P_M)$$

or

$$P_T = P_I + P_M - P_I P_M . \tag{8.1.2}$$

The probability of total information loss, which includes losses from both the servicing procedures and the symbol error, is shown in Figure 10. From Figure 10, it can be seen that for a given value of P_I, the probability for loss from servicing, the probability of total system loss, P_T, is then within 10% of P_I, if the probability of selecting the wrong symbol, P_M, is less than $.1P_I$. Referring to the curves of Figure 10, we can see that effort to reduce P_T below 1.1 P_I will be inefficient. The same result is also true if P_M is fixed and the variation of P_I considered. If $P_M = P_I$, then P_T will be approximately $2P_I$ for $P_I = P_M \leq .1$. Using these results, we are able to conclude that if either P_M or P_I is fixed by some physical restriction, optimization procedures or designers choice, the most reasonable value for the other (P_I or P_M), which was not fixed, is not less than one tenth the fixed value, and not more than equal to it. This results in a total loss probability, which is between twice and 1.1 times the fixed value. Efforts to reduce the probability of loss so that it approaches the fixed value more closely will be inefficient, and the probability of loss can not be less than the lowest value of P_I and P_M.

In the general case, achievable values of P_I and P_M will be related to cost, and the most economical design which can achieve a given total value of P_T may be obtained by minimizing the cost function that is associated with the particular system employed.

8.2. Criterion Functionals

The results of Chapter VI may be utilized to form criterion functionals for the purpose of obtaining optimal performance for a

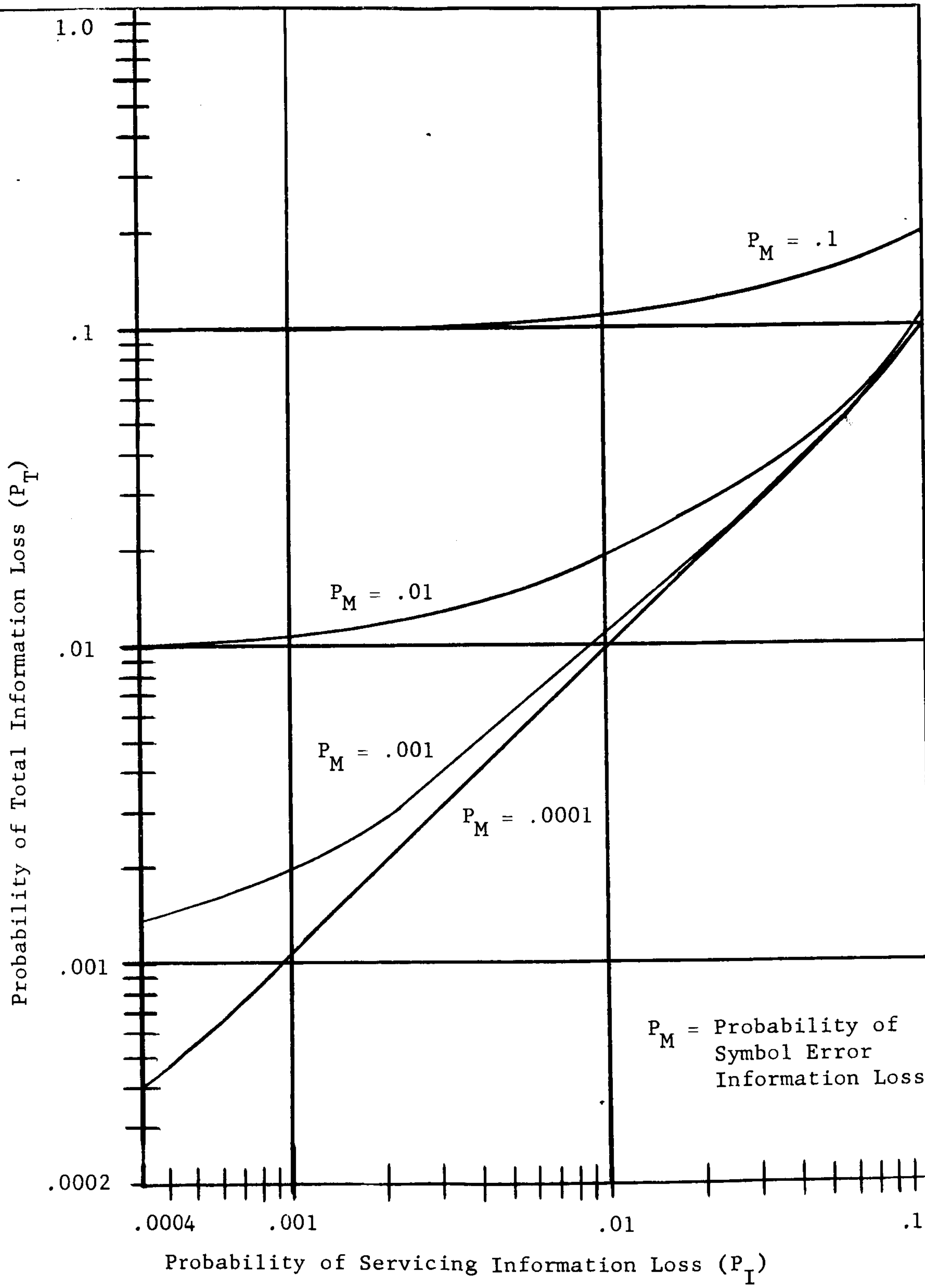

Figure 10. Probability of Total Information Loss

given system configuration, or for determining which of the possible system configurations gives a better performance subject to specific constraints.

Actually, the determination of criterion functionals is a subject for a complete study which can be best accomplished by the systems analyst after some requirements or limitations are imposed.

In this study, functionals may be proposed for illustrative purposes, but would hardly be assumed to actually conform to the desired results of a specific practical problem.

Two basic types of functionals are easily brought to mind. The first is a multiplicative type of functional where all the variables in the functional appear as functions of the variables in a multiplicative context. For instance, if bandwidth (BW) readout time (S), and probability of lost information (P_T) are the only factors to be considered, the functional F may be described by

$$F_1 = (S)(BW)(P_T)$$

Differences in relative importance of the factors might be accomplished by functionalization of the parameter; i.e., if readout time is more important than the other factors, we could use

$$F_1 = S^2 \cdot BW \cdot P_T, \text{ or} \tag{8.2.1}$$

$$F_1 = e^S \cdot BW \cdot P_T , \tag{8.2.2}$$

or a similar result.

Of course, once a criterion has been decided upon, the problem becomes to optimize F_1 by finding $\hat{S}$, $\widehat{BW}$, $\widehat{P_T}$ such that F_1 is an optimum (minimum for this example). This type of criterion functional has the disadvantages of being very sensitive to errors in specifying a satisfactory relationship of the parameters, and of being difficult to optimize because of the inherent non-linearity, subject to constraints.

An alternate functional is the additive cost functional, where each parameter and the associated cost is combined linearly. If C_S is the cost per unit time of readout time, C_B is the cost per unit bandwidth, and C_p the cost for each unit of probability, we may use

$$F_2 = (C_S)(S) + C_B(BW) + C_p P_T \ . \tag{8.2.3}$$

This type of formulation has the advantage of simplicity in optimization, but the disadvantage of assuming a linear cost relationship for each parameter.

8.3. Sequential Interrogation

Before we investigate the sequential interrogation case, we shall first require some definitions and symbols.

Let:

T_I = the interrogation or identification word lengths,

T_M = time for complete transmission of the source,

N = number of sources,

S = readout time for each source to transmit at least once,

$\alpha_s = S/NT_t$ or S/NT_M ,

$\gamma_s = S\lambda/N$,

T_t = total time for interrogation and response, and

t_D = delay between interrogation and response.

When each source is separately keyed on so that no overlap of data transmission times can occur, the probability of loss due to receiver overloading is, of course, zero. Also, assuming the source and the interrogator have the same symbol time (T),

$$T_M = T_s \, , \tag{8.3.1}$$

$$BW = \frac{2n_s}{T} \, , \tag{8.3.2}$$

$$T_t = T_M + t_D + T_I \, , \tag{8.3.3}$$

$$S = NT_t \, , \text{ and} \tag{8.3.4}$$

$$\lambda = \frac{1}{T_t} \quad . \tag{8.3.5}$$

8.4. Random Arrivals not Separable

Since the arrivals are not separable, along with the data transmission, each source must transmit an identification code so that the source may be uniquely identified.

Hence,

$$T_M = T_I + T_S \; . \tag{8.4.1}$$

Since the sources transmit randomly, the total readout time is greater than NT_M. It will depend on the range of the allowable time of transmission of each source. Therefore, we must define

$$S = \alpha_s \; NT_M \; , \tag{8.4.2}$$

where $\alpha_s > 1$. Let $\gamma_s N$ be the total number of transmission times that have occurred in S, hence, the arrival rate is

$$\lambda = \frac{\gamma_s N}{S} \; . \tag{8.4.3}$$

Since the sources are not separable, they must operate on the same frequency set, hence,

$$BW \doteq \frac{2n_s}{T} \; . \tag{8.4.4}$$

The probability of losing information due to overlapping transmissions is given in 6.1.6 and is reproduced here as 8.4.5. P_I is given as

$$P_I = 1 - e^{-2\lambda T_M} \; . \tag{8.4.5}$$

8.5. Random Arrivals with Separability

Again, we have,

$$S = \gamma_s N/\lambda \text{ and} \tag{8.5.1}$$

$$T_M = KT_s \ . \tag{8.5.2}$$

The bandwidth depends on whether the signals are separable by frequency or not. If separable by frequency,

$$BW = 2n_s N/T \ , \tag{8.5.3}$$

if not,

$$BW = 2n_s/T \ . \tag{8.5.4}$$

The probability of lost information on each arrival depends upon the system being considered, and will be discussed later.

8.6. Cost Criterion Functional

Let:

C_I = the cost of the interrogation capability,

C_R = the cost of each receiver,

C_K = the cost of each data period per message,

C_U = the cost of obtaining the separability capability other than by frequency, and

C_B, C_S and C_p are defined as before.

Then

$$F_2 = C_I\Delta + (BW)C_B + SC_S + c\ C_R + KC_K + P_TC_p + \epsilon C_U \cdot \tag{8.6.1}$$

Δ and ϵ are either 0 or 1, depending on the applicability of C_I and C_U in the system under consideration. The cost functional may be used in two different ways. The first is to maximize the performance of a given system. The second is to select the better system from among the various possibilities.

Consider the system where the arrivals are random, but not separable. Then if $P_M \leq .1\ P_I$ and $P_T = P_I$,

$$F_b = \frac{2n_s}{T} C_B + \frac{\gamma_s NC_S}{\lambda} + C_R + C_K + C_p\ (1-e^{-2\lambda T_M}) \ . \tag{8.6.2}$$

Let:

$\gamma_s = 2,$

$N = 5000,$

$C_S = 2,$

$T_M = 1,$ and

$C_p = 10^5.$

To find $\widehat{\lambda}$ such that F_b is a minimum, we need only find $\frac{dF_b}{d\lambda} = 0$. This results in the following expression:

$$\lambda^2 e^{-2\lambda} = .1 . \qquad (8.6.3)$$

Hence,

$$\lambda = .55 \quad \text{for a minimum, since} \quad \frac{d^2F_b}{d\lambda^2} > 0.$$

Next, consider the case where we wish to find the best system among the various possibilities.

The cost functionals for sequential interrogation become

$$F_a = \frac{2n_s}{T} C_B + NT_tC_S + C_K + C_R + C_I, \qquad (8.6.4)$$

for random arrivals without separability,

$$F_b = \frac{2n_s}{T} C_B + \alpha_s NT_MC_S + C_K + C_R + P_TC_p, \qquad (8.6.5)$$

for random arrivals with separability by frequency,

$$F_c = \frac{2n_sNC_B}{T} + \frac{N\gamma_sC_S}{\lambda} + C_K + c\ C_R + P_TC_p, \qquad (8.6.6)$$

and

$$F_d = \frac{2n_s NC_B}{T} + \frac{N\gamma_s C_S}{\lambda} + KC_K + C_R + P_T C_p \ . \qquad (8.6.7)$$

For random arrivals with separability by other than frequency,

$$F_e = \frac{2n_s C_B}{T} + \frac{N\gamma_s C_S}{\lambda} + C_K + cC_R + P_T C_p + C_U, \qquad (8.6.8)$$

and

$$F_t = \frac{2n_s C_B}{T} + \frac{N\gamma_s C_S}{\lambda} + KC_K + C_R + P_T C_p + C_U. \qquad (8.6.9)$$

Let the following parameters take on the specific values as shown.

$T_t = 93$	$C_p = 10^4$
$T_M = 83$	$C_B = 1$
$T_s = 70$	$C_S = 1$
$N = 5000$	$C_K = 300$
$\gamma_s = 2$	$C_R = 400$
$n_s = 2$	$C_I = 10^7$
$P_I = .1$	$C_U = 10^7$
$P_M = .1P_I$	

Then using the above, we can find

$F_a = 10{,}007{,}623,$

$F_b = 202{,}332$ with $\lambda = .05,$

$F_c = 1{,}412{,}500$ with $\lambda = 1.0$, $c = 3$, $K = 1$, dependent,

$F_d = 1{,}415{,}100$ with $\lambda = 1.0$, $c = 1$, $K = 4$,

$F_e = 10{,}012{,}780$ with $\lambda = 1.0$, $c = 3$, $K = 1$, dependent, and

$F_f = 10{,}015{,}380$ with $\lambda = .8$, $c = 1$, $K = 4$.

Using this criterion and with these values, one would select F_b as the minimum and, hence, the optimum system solution would be to use random arrivals without separability. The situation is altered if the same values are specified, except that $P_I = .001$ instead of $P_I = .1$. In this case,

$F_a = 10{,}007{,}623,$

$F_b = 10{,}001{,}342$ with $\lambda = .001,$

$F_c = 1{,}427{,}500$ with $\lambda = .8$, $c = 3$, $K = 1$, dependent,

$F_d = 1{,}427{,}600$ with $\lambda = .8$, $c = 1$, $K = 4$,

$F_e = 10{,}013{,}390$ with $\lambda = .8$, $c = 3$, $K = 1$, dependent, and

$F_f = 10{,}013{,}490$ with $\lambda = .8$, $c = 1$, $K = 4$.

For this situation, the minimum is F_c, although F_d is about the same. The system selected would use random arrivals with frequency separation with either $c = 3$, $K = 1$, dependent receiver operation, or $c = 1$, $K = 4$ and $\lambda = .8$.

While it must be obvious that these examples are somewhat contrived, they are only meant to demonstrate how the results derived in the text can be applied to systems decisions. In an actual system to be designed, those restrictions and specifications attending its utilization will be required to perform optimization or decisional processes. They, of course, will differ in value from those assigned here. However, the methods described here should be applicable.

IX. DISCUSSION

In order to consider the major contributions to the problem of information loss in a large information collection system, it was necessary to consider the information lost by an erroneous decision as to what symbol was actually sent, in addition to information lost by incompletely serviced or lost arrivals.

For information lost by erroneous symbol interpretations, we assumed that the channel was a noisy, fading medium. The noise was assumed to be Gaussian white additive noise and the fading variable was assumed to follow a Rician distribution. The advantage of the Rician distribution is that it is more general than the normally assumed Rayleigh fading variable, and by the proper selection of constants can be made to reduce to a constant fading, Rayleigh fading, mixed fading, or approach the Gaussian fading distribution.

A maximum likelihood type decision process resulted in the receiver shown in Figure 8 and a symbol error probability given by equation 7.3.30. The special case of binary symbols is given in equation 7.3.31 and is shown in Figure 9. For values of g (ratio of energies in the random to specular channels) which are less than 1, the probability of error falls off faster than for $g \geq 1$, where the signal to noise ratio for the specular channel (ρ) is greater than 3. For values of $g \leq .01$, the probability is independent of g for the range shown in Figure 9 and expressed by 7.3.31. For very large values of g and small values of ρ, the probability approaches the result in 7.3.34.

Equation 8.1.2 and Figure 10 show that the total information loss probability for ordinary use would be between 1.1 and 2 times greater than the largest value of P_I and P_M, if both are $\leq .1$. Attempts to reduce the probability of total loss below 1.1 times the largest value are very inefficient.

For information lost because the serving system is overloaded with arrivals, it was assumed that a master scanning receiver could find, assign, and keep track of each new arrival and past arrivals, if necessary, so that information is not lost by the scanning master receiver. This assumes a memory capability associated with the master scanner which keeps track of each arrival, new or old, as time progresses and as each arrival leaves the system, whether serviced completely or not. It also might imply that each arrival has a short interval of transmission designed to allow the master scanner to find it before the actual data begins, but this interval would be small compared with a service period.

The slave receivers are assumed to be capable of being assigned to any potential arrival and the time required for complete servicing is fixed and uniform for each arrival. This can be justified by the telemetry type of data considered and the fact that each source samples the same environmental parameters. The large number of sources following the same type of probability distribution which determines when each source transmits, and the assumption of transmission independence leads to the Poisson distribution of arrivals.

However, should dependence of only a small number of sources occur, we would still have a close approximation to the actual process with perhaps a different value of mean arrival rate.

If data redundancy is considered whereby the transmission of each source contains several identical data periods, we are able to note from Figure 6 that for the higher values of λT_s, the probability does not fall off very fast, and a low probability of lost information due to receiver overloading can only be accomplished by the addition of many data periods. The analytical expression developed which describes this case is given by equation 6.2.44. The advantage of data redundancy is that if upon arrival the server is found to be occupied, the signal can "wait" several service periods for the server to become free. If the number of data periods per arrival is greater than the necessary waiting time by at least a service period, then a complete service can still be performed. The data redundancy capability may be useful where a systems modification of an existing system is required to accommodate more sources, or to reduce the losses, and the reception devices are not readily accessible, such as with an orbiting satellite. This means that the sources are required to be controllable. It can also be noted in Figures 2, 3, 4, and 6 that the relative efficiency of adding more redundant data periods is less than adding more servers.

Consider, now, the case where additional servers are used in a service discipline where no calls are lost, but they may be incompletely serviced. We can find from Figure 2, that the probability of an incomplete service becomes small faster than for redundant data periods, especially for large values of λT_s. The formulation describing this performance is given in equation 6.2.3. It can be noted from Figure 2 that larger values of λT_s may be used ($\lambda T_s \geq 1.$) with a few servers with a satisfactory probability of loss. This is not true for the results shown in Figure 6. This type of serving discipline is useful because every arrival is eventually assigned to some slave and processed at least partially. This type of discipline is easier to implement in the sense that every signal is accounted for and no record keeping is required. The disadvantage is, of course, that arrivals that can not be completely processed begin service, and, hence, their time in service is wasted.

The advantage of additional servers is obvious. The more servers, the smaller the probability of loss becomes. The cost and ease of implementation, however, are factors that must be considered before deciding whether to add servers or data periods.

In the case where arrivals which can not be serviced immediately are lost, we have two types of operation, with either independent or dependent servers.

With independent servers every c^{th} arrival, where c is the number of servers, is channeled to the same server. The call is serviced completely if that server is free upon arrival and lost otherwise. The disadvantage of independent operation is that a particular server may still be occupied upon the arrival of its next call and the new call would be lost, while some other server might be free and could, in theory, handle the call that was lost. The advantage is, of course, that assigning the arrivals is less complex and easier to implement.

The results of this type of servicing operation are shown in Figure 3 and described by equation 6.2.13. From Figure 3, we can see that the results are similar to Figure 2.

If dependent servers are utilized as shown in Figure 4, we can note that a small advantage is gained over independent operation. Because the advantage is small, we may use equation 6.2.13 to approximate the result, and to act as an upper limit for the probability, also.

The remaining type of operation is shown in Figure 1. This is a single server case where the arrivals are random, but not separable except by time. As expected, an increasing arrival rate causes a larger information loss. Equation 6.1.6 describes the result. It can be seen in Figure 1, that for a lower value of probability of loss, λT_s must be very small. The advantage of such a system is simplicity, but the disadvantage is that for reasonable losses, λT_s must be very small.

A comparison of the loss probabilities for each system, for specific values of c, K, and λT_s, is shown in Table 1. The various systems may be compared for relative efficiency in reducing information loss. Case 3, where dependent receivers are considered for the delayed calls lost servicing routine, has the lowest probability of loss; Case 2 has the second best, and Case 1 the next. These are followed by redundant data and the non-separable case in decreasing order of complexity and, hence, decreasing order of improvement. In the situation where there is only one receiver and one data period, Case 1 and the redundant data case reduce to the same system. Likewise, Cases 2 and 3 are identical since no distinction can be made for dependent and independent operation with only one server. Referring to Table 1, we can note that redundant data operation does provide an effective way of lowering the probability of loss, provided that the arrival rate is $< .8$. For values of λT_s greater than .8, the system becomes saturated with unserviced arrivals and further improvement is very inefficient in terms of additional data periods. For only a few receivers, there is a greater difference between Case 1, where all calls start service, and Cases 2 and 3, where delayed calls are lost, than for larger values of the number of receivers, where the curves approach each other.

Next, consider the time-bandwidth problem for each type of operation. Time is intended to mean the time for a complete readout of the entire set of sources. When they operate probabilistically, we shall imply that every source must transmit at least once in some finite interval which is part of the designers choice. Of course, whether or not the data from each source is completely serviced is a random determination, and the rate of loss dependent on the selection of λ, K, c, and the type of operation. Bandwidth shall mean the entire bandwidth for the operation.

If the sources are interrogated sequentially, then all sources respond on the same frequency. The bandwidth is just the approximate bandwidth for each symbol, $BW = \frac{2n_s}{T}$. The total bandwidth depends upon whether or not the interrogation signal is entirely within this band, or separate or wider. The time for readout without loss is, of course, just $N\,T_I$, where N is the number of sources and T_I includes interrogation time, delay time, and data transmission time.

Table 1

Comparison of Probability of Information Loss for Each System

System	c	K	Mean Number of Arrivals in One Service Period (λT_S) .3	.5	.8	1.0	3.0
Non-separable	1	1	.45	.64	.798	.87	.998
Case 1	1	1	.27	.38	.45	.55	.94
Case 2	1	1	.23	.33	.43	.50	.75
Case 1	3	1	.0035	.014	.045	.087	.58
Case 2	3	1	.0035	.014	.045	.075	.43
Case 3	3	1	.0031	.011	.039	.062	.36
Redundant Data	1	3	.0055	.035	.16	.30	.94
Case 1	5	1	.000016	.00017	.0014	.0035	.18
Case 2	5	1	.000016	.00017	.0014	.0035	.16
Case 3	5	1	.000015*	.00013*	.00125	.0032	.11
Redundant Data	1	5	.0001	.003	.06	.18	.94
Case 1	8	1	X	X	X	X	.012
Case 2	8	1	X	X	X	X	.011
Case 3	8	1	X	X	X	X	.010
Redundant Data	1	8	X	.00008	.0155	.13	.94

Case 1 is the case where all calls start service. Case 2 is the independent server case where delayed calls are lost. Case 3 is the dependent case where delayed calls are lost.

* denotes extrapolated values.

c is the number of servers.

K is the number of data periods.

X denotes $P_I \leq .00001$.

If the sources are operating randomly on the same frequency without separability, we again have $BW = \frac{2n_s}{T}$. However, the readout time may be quite extended since it depends on the rate of arrival and the upper time limit of the probability density that controls the source time of transmission. The arrival rate is required to be small if the loss is small, and then, the readout time may be very large $(S > \frac{N}{\lambda} > NT_s)$.

In the case of random arrivals with separability, the arrival rates can be made large by expending resources for the addition of servers or data intervals, and, hence, the readout times may be consequentially reduced. However, the bandwidth depends upon how separability is obtained. If the separability is by frequency then

$$BW = \frac{2n_s N}{T}.$$

In the case of random arrivals with separability, we note that it is more efficient, in terms of probability of lost information, to increase the number of receivers than to increase the number of data periods per message. While this may be true, it depends upon the relative costs of these alternatives to determine which would be more useful. The redundant data interval remains a useful technique for providing a lower probability of information loss due to receiver loading by the arrivals. If additional sources are added to the system, the effects of them may be partially or completely compensated by the addition of extra data periods or receivers.

In determining what type of separability a system should utilize, it is obvious that, should frequency be chosen, the attendent bandwidth is linearly increased with the number of sources. Location separability implies a concept whereby large antennas and high frequencies are required. Large antennas in space may not be practical, and the problem of directivity may also not be practical because of motion or stability at either end of the propagation link.

We have used FSK and its inherent orthogonal nature for the symbols of a message, but it is possible to utilize this technique of orthogonality for the entire message. Such reception and isolation would be accomplished by the use of receivers performing as correlators or matched filters. However, any utilization of amplitude or phase as a factor in the received signal to be correlated with stored signals would be subject to variation of the propagating medium, and, if the number of sources is expected to be large, it would seem likely that some frequency orthogonality would be required in any event.

The curves for delayed arrivals lost, dependent receivers and independent receivers, do not show a very large difference, and both are not very different from the curves where each call starts service. From this, it would seem that the most applicable choice of the reception subsystem would be the one which is easiest to implement. This, of course, depends on the exact handling of arrivals by a master scanning unit. If delayed arrivals are lost, then the master scanner must have a memory device for reference as each signal is encountered, to determine whether it was present on the last scan, or not. Arrivals are assigned to a slave receiver for service only if a new signal appears and a slave is free. In the case where each call receives service (possibly incomplete), a memory is also needed, the calls being assigned to a particular slave receiver sequentially as they are found, but not serviced until the receiver is free.

Perhaps the most interesting type of operation is the use of data redundancy. This is similar to a customer arriving at a service facility and, perhaps, encountering a waiting line. If he must wait too long in line, he can only be incompletely serviced. However, the effect of the redundant data periods is to allow a finite amount of waiting time for each arrival with the result that a complete service is still possible.

The net effect of extra receivers and data periods is that by expending our resources in them, we can increase the arrival rate to acceptable values of readout time while maintaining a satisfactory information lost performance.

The effect of symbol error has, of course, been treated in this study as a separate and independent event. It, of course, depends on the number of states per symbol, the signal to noise energy ratio, and other factors of the fading environment. It is necessary that this probability be calculated for a complete viewpoint of the probability of information lost problem. Probability of symbol error was considered to be just a noise and fading problem, and when interference must be considered, it was considered as part of the problem of receiver loading.

Several examples are given in Chapter VIII which demonstrate how one might use the performance equations and curves found in Chapters VI and VII. In principle, we can optimize within a given system operation by finding the optimum solution to the criterion functional for that system, or by comparing the various functionals associated with each type of system operation, we may select that system which gives the best performance. Best performance in this context is, of course, defined in terms of the given criterion functional which is valid for the particular situation.

The examples given in Chapter VIII are intended for illustrative purposes only. They may represent simple typical solutions and criteria, but any specific practical situation will normally provide certain unique problems and evaluations that must be considered separately by the potential user.

In this report several computer simulations were required to check the results of analytical data. In all cases, a statistical sample of 10,000 trials was made using an IBM 7074. Then the sample arrivals were processed by a program simulating the particular serving system. The net result was that out of 10,000 arrivals, some were completely processed, and some were incompletely processed or lost entirely. An example of these programs is given in Appendix B. The probability of losing information for any arrival was calculated from the number of lost or incompletely processed arrivals. Most of the data checks very closely with analytical data, thus verifying the accuracy of the results. Small deviations that are noticeable between analytical and simulated results occur for small values of probability. This is expected since, for these data points, out of 10,000 arrivals only a few are not completely processed, and the graphs are very sensitive to a small deviation from the sample mean.

In order to simulate Poisson arrivals, it was necessary to convert the random number generated by the computer to a random number following the negative exponential probability distribution. The generated number was converted to represent the time between arrivals. The conversion of the uniform random number to a negative exponential is shown in Appendix A.

In Chapter V we discussed several applications for which this type of system performance analysis might be useful.

In the first system, we have assumed a traffic density reporting model for use in control of traffic flow. This type of system would probably have the basic requirement of simplicity, and because of its more local nature, all of the reporting elements and collection devices would be readily available. For such a system, telephone lines or the equivalent would probably be used instead of atmospheric propagation. In this event, the signal to noise ratio could be sufficiently large and fading for all practical purposes non-existent. It would seem likely that such a situation would imply the use of sequential interrogation, resulting in no loss to the system from servicing problems. The only loss would be from symbol errors, and this can be reduced to negligible amounts by the use of telephone lines and high signal to noise ratios. Additional reporting sites could be readily added to an existing system with the readout time the only parameter affected.

In the second application suggested, we have considered a weather reporting model. Such a system would in a large measure depend upon the amount of funds available and the potential use of the data collected. A simple model using sequential interrogation might be considered first. However, the system desired might be more sophisticated than this is if its uses are more general and additions to the number of reporting stations are possible. Any model covering large geographical areas would require reporting stations in the ocean and in uninhabited regions of the earth such as polar regions and deserts. This would require the use of atmospheric propagation and, hence, symbol errors due to fading and noise would have to be considered. Probably any practical use of weather reporting on a large geographic scale would result in several steps of sophistication. Perhaps, at first, a sequential interrogation system, later as the fine resolution of the model becomes more important and many more sources are added, we would use a more sophisticated system such as all calls starting service. Still later, as readout time becomes more important for short time analysis of the weather patterns, perhaps a dependent delayed calls lost system would be desirable.

In the third system, a reconnaissance system may be considered. In such a system, control of the data sources would not be possible. It is assumed for this model that we are not seeking content of the received signals, but merely measurements of parameters such as frequency, pulse width, modulation type, pulse repetition rate, location, etc. In order to accomplish these measurements adequately, we shall assume the signal must be received and processed for a fixed time interval. Then, if the signal duration is longer than the time required for processing, we have a situtation similar to the data redundancy case. For such a reconnaissance system, the data redundancy is already built in. The problem would reduce to minimizing the probability of loss from the servicing, since we would have no control of the fading and signal to noise ratios.

X. CONCLUSIONS

The assumption of a Rician fading distribution and additive white Gaussian noise leads to results represented by equation 7.3.30. Figure 9 shows the special binary case of equation 7.3.30. The graphs show that the curves for small values of the ratio (g) of energy from the random to energy from constant channels, the probability of error is independent of that ratio. For values of ρ, the signal to noise ratio in the fixed channel, which exceeds 3, the probability of error falls off faster for $g<1$ than for $g\geq 1$. For values of $g>>1$, the curves approach the approximation given by 7.3.34. The net effect of selecting the wrong symbol state affects the total system loss as shown in Figure 10 and expressed by equation 8.1.2. For most normal uses, the probability of the total system error would be 1.1 to 2 times the greater of the probability of loss from both the servicing and the symbol decision processes, provided that both are less than .1.

The effects of data redundancy on the probability of lost information is shown by equation 6.2.44 and Figure 6. The addition of data periods is not as efficient as adding servers. The curves in Figure 6 do not show a fall off as fast as those in Figures 2, 3, and 4. This is especially true for larger values of λT_s, where a low probability of loss can only be achieved by utilizing a considerable number of data periods.

Additional receivers may also be used to lower the probability of loss or to increase the arrival rate. These results are shown in

Figures 2, 3, and 4 and are expressed by equations 6.2.3 and 6.2.13. The curves are very similar, with the dependent servers having a slightly lower probability of information loss, if all the other parameters are equal. The curves of the independent servers with delayed arrivals lost and the curves where all arrivals start service approach each other for larger values of the number of servers. For lower values of the number of servers, the delayed calls lost servicing routine shows a slightly lower probability of loss.

For random arrivals without separability, Figure 1 and equation 6.1.6 describe the results. In this particular situation, we note that low probability of loss can only be accomplished for low values of arrival rate. This is a serious restriction if readout time is significant.

The bandwidths for each type of system considered may be approximately the same except for where frequency separation of the sources is required. The readout time is a function of the arrival rate, the number of sources, and the transmission time distribution. Usually the last two factors will be fixed by practical considerations and, therefore, the readout time will ultimately depend on the arrival rate. The effect of arrival rate on the probability of information loss has been shown in Figures 1, 2, 3, 4, and 6. By selecting K or c, λT_s and the probability of loss may be controlled to a satisfactory degree.

Further use of these performance results may be used in a search for optimum results using specific criterion functionals. Either a given system may be optimized or a decision as to which system alternative to choose may be accomplished by the proper use of these performance results and adequate criterion functionals.

The results for the total over-all system information loss probability are shown in Figure 10 and equation 8.1.2. This figure shows that the lowest possible probability of total loss (P_T) is equal to the lowest value of P_I and P_M, the respective probabilities of servicing loss and symbol error loss. If either P_I or P_M is set at some fixed value, the most reasonable value for the total probability of loss would be between 1.1 and twice the fixed value with the probability that is not set between .1 and 1 times that fixed value.

XI. SUGGESTIONS FOR FURTHER RESEARCH

Usually research into a few specific problems uncovers other problems and areas of research that appear to require further work. The present investigation is no exception. During the research, several other associated problems were encountered which were not considered in this work, but would seem appropriate for further attention.

Perhaps the most obvious area for further work is in the way servicing is accomplished. Naturally, combinations of multiple servers and redundant data intervals could be desirable, and the results of further work in this area would be interesting.

The use of priority signal handling might also be examined. The signals from certain sources might be considered to have a higher priority than the rest, and different service routines could be associated with each priority class. This could be of a pre-emptive nature where signals of a higher priority upon arrival could pre-empt signals of a lower class being serviced, or a head of the line discipline where, regardless of arrival time, higher priority signals go to the head of the line of lower priority signals. Priority class assignments could be on the basis of source location, or, perhaps, on the basis of whether or not the signal from each source was completely serviced on its previous arrival. If it was not completely serviced, then its priority class assignment could be increased. Further priority assignments for a continuous priority designation could be on the basis of time since last complete service, with the higher priorities going to the longer times.

There is also a great need for work in the area of performance criteria, cost-effectiveness, and the sensitivity of an optimal solution to parameter variation.

The use of satellites introduces certain problems which were not fully explored. The problems of reception that occur with satellite motion such as doppler shifting, and the moving satellite reception horizon have not been examined.

The tracking and handling problems that occur when the sources are free to move has not been examined, and seem to merit further attention.

The results of data redundancy for servicing and assignment operations that are different from that considered in this study could be examined. Calls delayed more than $(K-1)T_s$ could be lost, for instance, rather than incompletely serviced.

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APPENDIX A

Conversion of a Uniform Random Variable to a Negative Exponential Random Variable

Let:

Y_r = represent a random number from a uniform distribution such that $o \leq Y_r \leq Y_o$ and

X_r = represent a random number from an exponential distribution such that $o \leq X_r < \infty$.

Also, let

$$\int_o^{Y_r} \frac{1}{Y_o} \, dy = \int_o^{X_r} \lambda e^{-\lambda x} dx \; . \tag{13.1}$$

Then

$$\frac{Y_r}{Y_o} = 1 - e^{-\lambda X_r} \; . \tag{13.2}$$

Taking the natural log of both sides, we have

$$X_r = - \frac{1}{\lambda} \ln \left(1 - \frac{Y_r}{Y_o}\right). \tag{13.3}$$

In our particular case

$$Y_o = 1 , \tag{13.4}$$

therefore,

$$X_r = - \frac{1}{\lambda} \ln (1 - Y_r). \tag{13.5}$$

Hence, it is obvious then that

$$X_r \epsilon\ p(x) = \lambda e^{-\lambda x} \quad \text{for} \quad o \leq X_r < \infty \tag{13.6}$$

if

$$Y_r \epsilon\ p(y) = \frac{1}{Y_o} = 1 \quad \text{for} \quad o \leq Y_r \leq 1. \tag{13.7}$$

APPENDIX B

Daft Program for Simulation of Dependent Operation with Delayed Calls Lost

BEGIN DAFT SOURCE DECK

```
          DIMENSION F(40),X(40),A(10),K(10)
    1     FORMAT('1',T5,'N',T15,'C',T25,'A',T35,'IS',T55,'IL')
    2     FORMAT(T2,T5,T12,I5,T22,F5.2,T32,I5,T52,I5)
    3     FORMAT(I5)
    4     FORMAT(I2)
    5     FORMAT(F5.2)
    6     FORMAT(I10)
    7     FORMAT('0',T10,'BASE')
    8     FORMAT(T8,I10)
          I=0
          J=0
          READ 6,IB1
          CALL SETBASE(IB1)
          READ 3,M
          PRINT 1
   10     READ 4, IK
          IF(IK) 12,12,11
   11     J=J+1
          K(J)=IK
          GO TO 10
   12     J1=J
   13     READ 5,BA
          IF(BA) 15,15,14
   14     I=I+1
          A(I)=BA
          GO TO 13
   15     I1=I
          DO 100,I=1,I1
          DO 100,J=1,J1
          KK=K(J)
          N=0
          IS=0
          IL=0
          T1=0.
          DO 20,L=1,KK
          R1=RAND(.99999999)
          D1=1.-R1
          Z1=-(1./A(I))*ALOG(D1)
          T1=T1+Z1
          F(L)=T1
```

```
20      X(L)=F(L)+1.
        T=T1
24      IF(M-N) 50,50,25
25      R=RAND(.99999999)
        D=1.-R
        Z=-(1./A(I))*ALOG(D)
        T=T+Z
        C=T
        N=N+1
        L=1
26      IF(C-X(L) 28,27,27
27      IS=IS+1
        F(L)=C
        X(L)=F(L)+1.
        GO TO 24
28      IF(KK-L) 30,30,32
30      IL=IL+1
        GO TO 24
32      L=L+1
        GO TO 26
50      PRINT 2,N,K(J),A(I),IS,IL
100     CONTINUE
        PRINT 7
        CALL SAVEBASE(IB)
        PRINT 8,IB
        STOP
END DAFT SOURCE DECK

DATA CARDS
6073979627
10000
01
02
03
04
05
06
07
08
10
BLANK
00.30
00.50
00.80
01.00
03.00
BLANK
```

The Daft programming language is very similar to Fortran. Several routine functions appear in this program that might be explained briefly.

RAND - This is a quasi-random number generator. The number is selected from a uniform distribution between 0 and .99999999.

SET BASE - This is a routine to set the base number of the random number generator in order that the same quasi-random numbers are not repeated.

SAVEBASE - This is a routine that can produce a printout of the last base number of the random generator.